Bearing Capacity of Shallow Foundations

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The problems of soil mechanics can be divided into two principal groups

stability problems

elasticity problems

Karl Terzaghi, 1943

foundations are generally grouped into two categories:

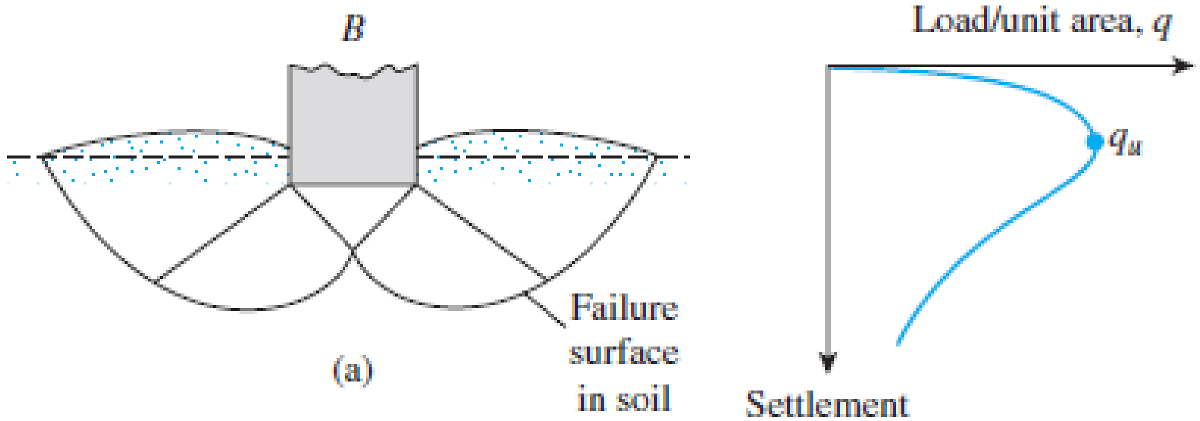
foundations are designed to transmit load from the structure they support to the soil

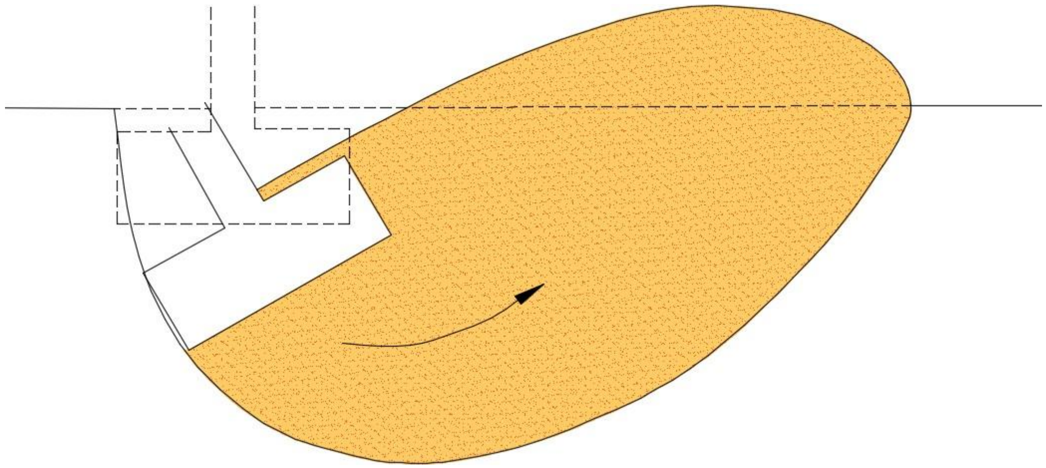
1.Shallow Foundations

2. Deep Foundations

modes of failure

General Shear Failure





The following are some characteristics of general shear failure:

✓Occurs over dense sand or stiff cohesive soil.

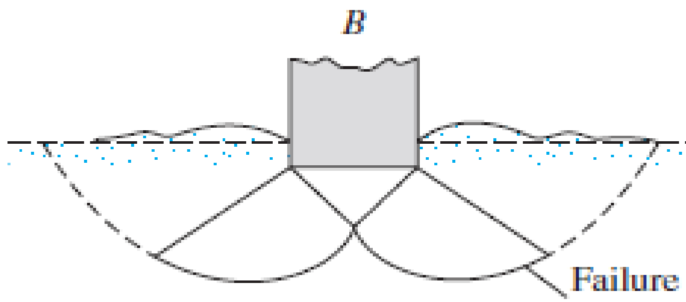
✓Involves total rupture of the underlying soil.

✓ For general shear failure, the ultimate bearing capacity has been defined as the bearing stress that causes **a sudden** catastrophic failure of the foundation.

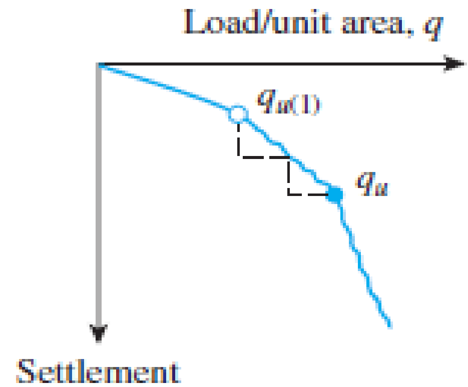
✓ As shown in the above figure, a general shear failure ruptures occur and pushed up the soil on both sides of the footing (In laboratory).

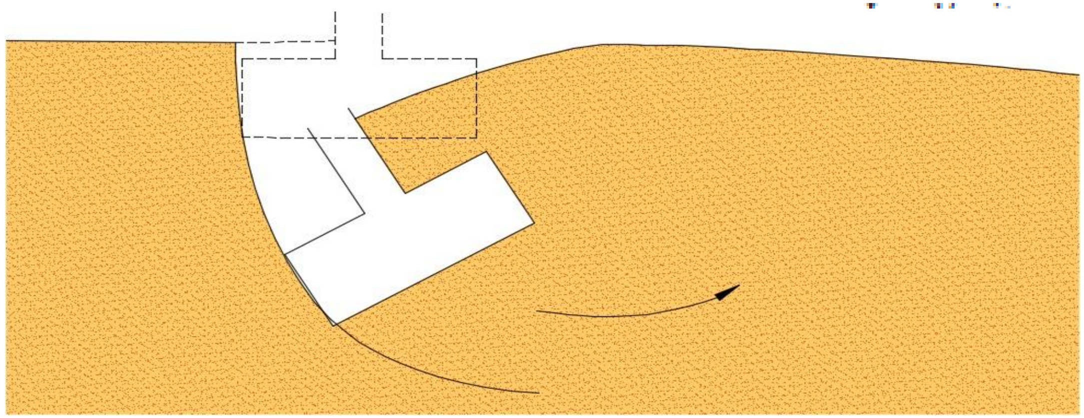
However, for actual failures on the field, the soil is often pushed up on **only one side** of the footing with **subsequent tilting** of the structure

Local Shear Failure



(b)





The following are some characteristics of local shear failure:

✓Occurs over sand or clayey soil of medium compaction.

✓Involves rupture of the soil only immediately below the footing.

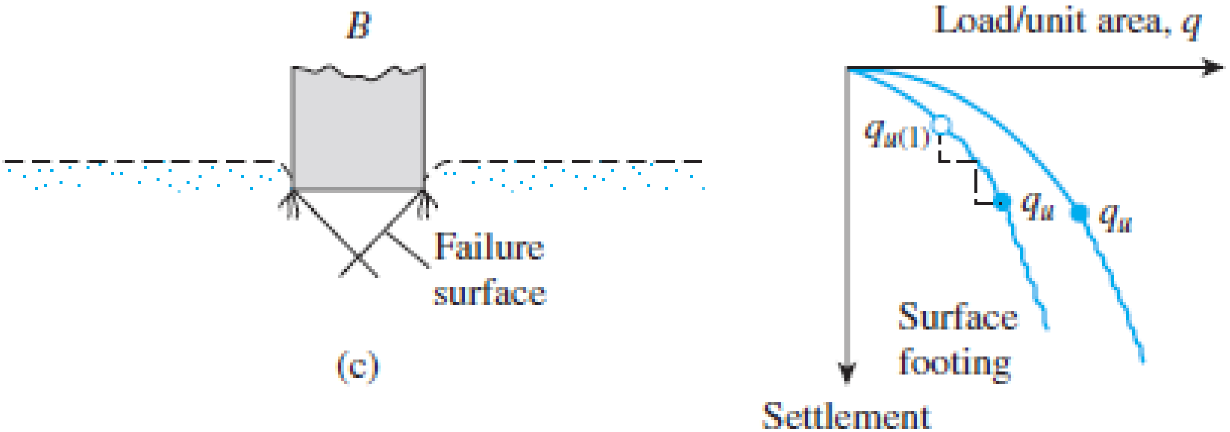
✓There is soil bulging on both sides of the footing, but the bulging is not as significant as in general shear. That's because the underlying soil compacted less than the soil in general shear.

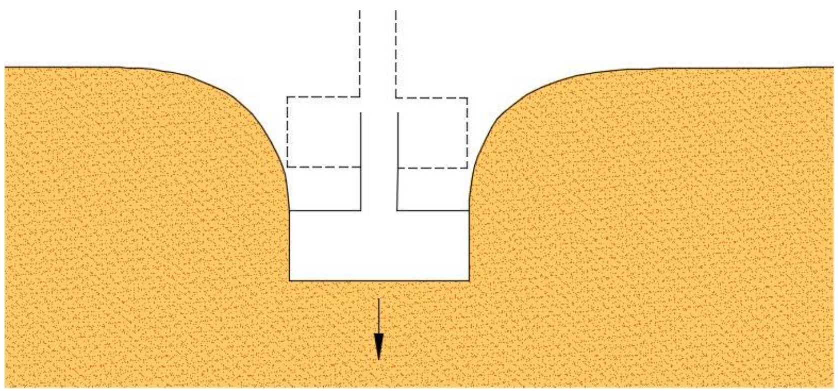
The failure surface of the soil will **gradually** (not sudden) extend outward from the foundation (not the ground surface) as shown by **solid lines**

✓Because of the transitional nature of local shear failure, the ultimate bearing capacity could be defined as the firs failure load ($q_{u,1}$) which occur at the point which have the first measure nonlinearity in the load/unit area- settlement curve, or at the point where the settlement starts rabidly increase .

✓In this type of failure the value of (q_u) it's not the peak value so, this failure called (Local Shear Failure).

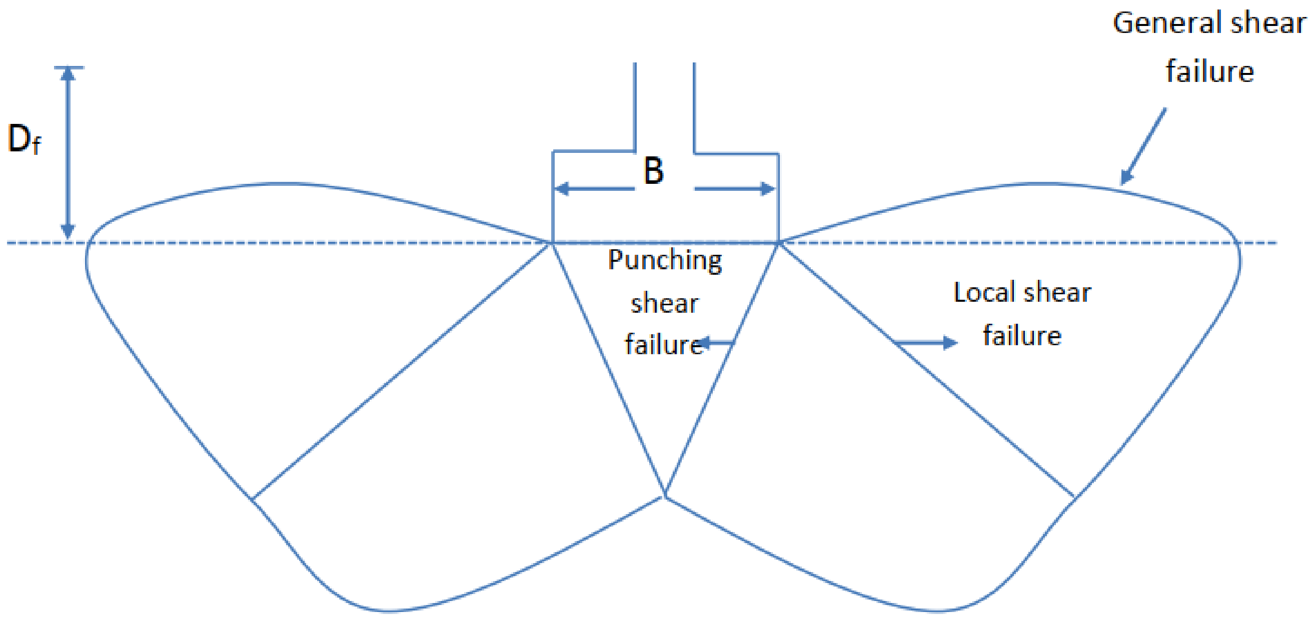
Punching Shear Failure





The following are some characteristics of punching shear failure:

- ✓ Occurs over fairly loose soil.
- ✓The soil outside the loaded area remains relatively uninvolved and there is a minimal movement of soil on both sides of the footing.
- ✓The process of deformation of the footing involves compression of the soil directly below the footing as well as the vertical shearing of soil around the footing perimeter.
- ✓Beyond the ultimate failure (load/unit area) ($q_{u,1}$), the (load/unit area)-settlement curve will be steep and practically linear.


General shear
failure
Df
B
Punching
shear
failure
Local shear
failure

Ultimate Bearing Capacity (q_u)

It's the **minimum load per unit area** of the foundation that causes shear failure in the underlying soil.

Allowable Bearing Capacity (q_{all})

It's the load per unit area of the foundation can be resisted by the underlying soil without any unsafe movement occurs (shear failure) and if this load is exceeded, the shear failure **will not** occur in the underlying soil **till** reaching the ultimate load.

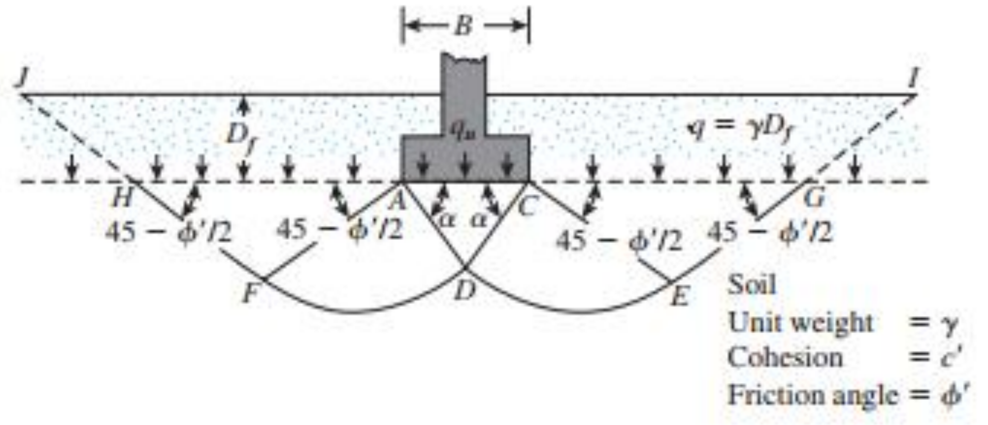
Terzaghi's Bearing Capacity Theory

1. The foundation is considered to be shallow if ($D_f \le B$).

2. The foundation is considered to be strip or continuous if ($B/L \to 0.0$). (Width to length ratio is very small and goes to zero), and the derivation of the equation is to a strip footing.

3. The effect of soil above the bottom of the foundation may be assumed to be replaced by an equivalent surcharge ($q = \gamma \times D_f$). So, the shearing resistance of this soil along the failure surfaces is neglected (Lines ab and cd in the below figure)

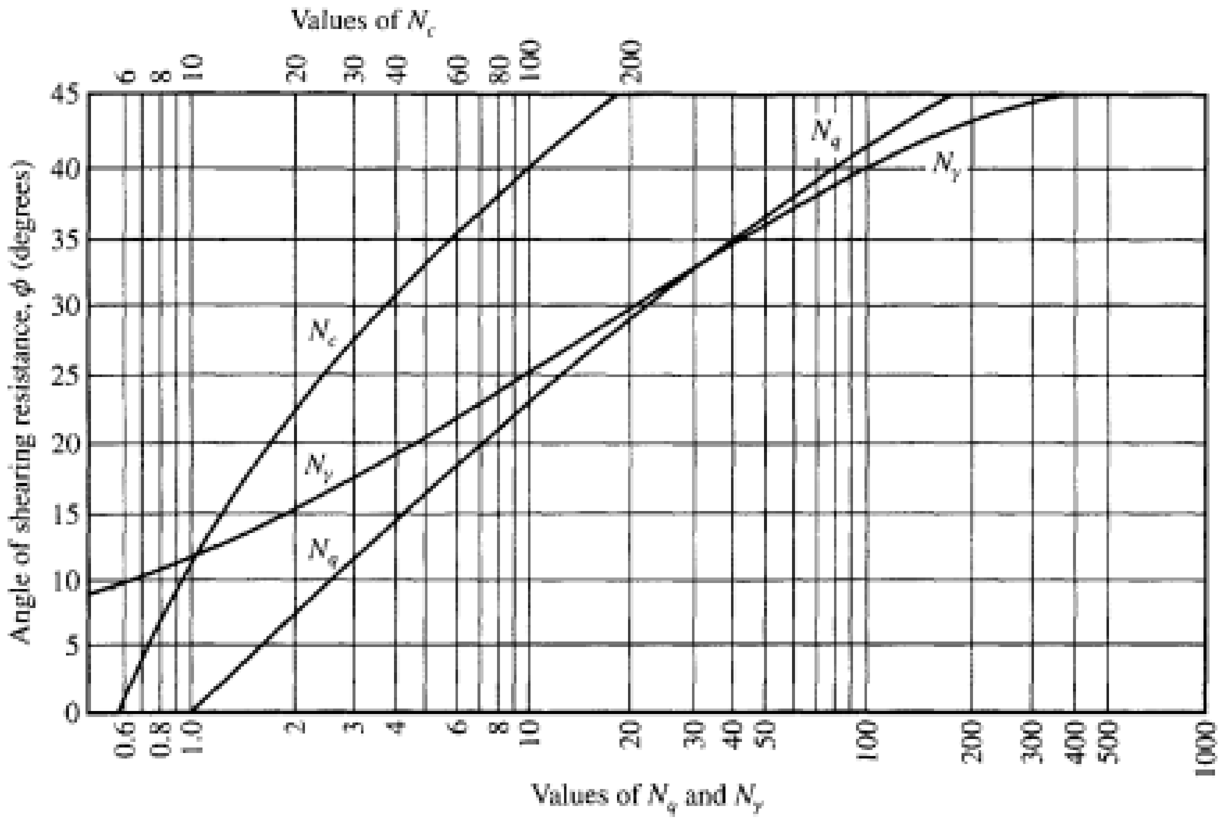
4. The failure surface of the soil is similar to general shear failure (i.e. equation is derived for general shear failure)



$q_u = cN_c + qN_q + 0.5B\gamma N_\gamma$ (for continuous or strip footing)

$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$ (for square footing)

$q_u = 1.3cN_c + qN_q + 0.3B\gamma N_\gamma$ (for circular footing)



Values of N_c
6
8
10
20
30
40
60
80
100
200
Angle of shearing resistance, ϕ (degrees)
45
40
35
30
25
20
15
10
5
0
N_c
N_γ
N_q
N_q
N_γ
0.6
0.8
1.0
2
3
4
6
8
10
20
30
40
50
100
200
300
400
500
1000
Values of N_q and N_γ

ϕ' (deg)	N_c	N_q	N_γ^a	ϕ' (deg)	N_c	N_q	N_γ^a
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	16.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	116.31
16	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	161.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	416.14	1072.80
25	25.13	12.72	8.34				

TABLE (1)

local shear failure

$$q_u = \frac{2}{3} cN'_c + qN'_q + 0.5B\gamma N'_\gamma \quad \text{(for continuous or strip footing)}$$

$$q_u = 0.867cN'_c + qN'_q + 0.4B\gamma N'_\gamma \quad \text{(for square footing)}$$

$$q_u = 0.867cN'_c + qN'_q + 0.3B\gamma N'_\gamma \quad \text{(for circular footing)}$$

N'_c, N'_q, N'_γ = Modified bearing capacity factors

(1)

ϕ' (deg)	N'_c	N'_q	N'_γ	ϕ' (deg)	N'_c	N'_q	N'_γ
0	5.70	1.00	0.00	26	16.53	6.05	2.59
1	5.90	1.07	0.005	27	16.30	6.54	2.88
2	6.10	1.14	0.02	28	17.13	7.07	3.29
3	6.30	1.22	0.04	29	18.03	7.66	3.76
4	6.51	1.30	0.055	30	18.99	8.31	4.39
5	6.74	1.39	0.074	31	20.03	9.03	4.83
6	6.97	1.49	0.10	32	21.16	9.82	5.51
7	7.22	1.59	0.128	33	22.39	10.69	6.32
8	7.47	1.70	0.16	34	23.72	11.67	7.22
9	7.74	1.82	0.20	35	25.18	12.75	8.35
10	8.02	1.94	0.24	36	26.77	13.97	9.41
11	8.32	2.08	0.30	37	28.51	16.32	10.90
12	8.63	2.22	0.35	38	30.43	16.85	12.75
13	8.96	2.38	0.42	39	32.53	18.56	14.71
14	9.31	2.55	0.48	40	34.87	20.50	17.22
16	9.67	2.73	0.57	41	37.45	22.70	19.75
16	10.06	2.92	0.67	42	40.33	25.21	22.50
17	10.47	3.13	0.76	43	43.54	28.06	26.25
18	10.90	3.36	0.88	44	47.13	31.34	30.40
19	11.36	3.61	1.03	45	51.17	35.11	36.00
20	11.85	3.88	1.12	46	55.73	39.48	41.70
21	12.37	4.17	1.35	47	60.91	44.54	49.30
22	12.92	4.48	1.55	48	66.80	50.46	59.25
23	13.51	4.82	1.74	49	73.55	57.41	71.45
24	14.14	5.20	1.97	50	81.31	65.60	85.75
25	14.80	5.60	2.25				

(2)

$$\phi_{modified,general} = \tan^{-1}\left(\frac{2}{3}\tan\phi_{local}\right)$$

For example: Assume we have local shear failure and the value of $\phi = 30^{\circ}$

1. By using Table (2) $N_c', N_q', N_\gamma' = 18.99, 8.31, \text{and } 4.9$ respectively
2. By using Table (1) $\phi_{modified,general} = \tan^{-1}\left(\frac{2}{3}\tan 30^{\circ}\right) = 21.05^{\circ} \rightarrow$

$N_c', N_q', N_\gamma' = 18.92, 8.26, \text{and } 4.31$ respectively

General Bearing Capacity Equation

(Meyerhof Equation)

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

F_{cs}, F_{qs}, $F_{\gamma s}$ = Shape factors

F_{cd}, F_{qd}, $F_{\gamma d}$ = Depth factors

F_{ci} , F_{qi}, $F_{\gamma i}$ = Inclination factors

Shape Factors:

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

Depth Factors:

➢ For $\frac{D_f}{B} \leq 1$

1. For $\phi = 0.0$

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

2. For $\phi > 0.0$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\textbf{radians}}$$

$$F_{\gamma d} = 1$$

➢ For $\frac{D_f}{B} > 1$

1. For $\phi = 0.0$

$$F_{cd} = 1 + 0.4\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

2. For $\phi > 0.0$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$$

$$F_{\gamma d} = 1$$

Inclination Factors:

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^{\circ}}{90}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta^{\circ}}{\phi^{\circ}}\right)$$

β° = Inclination of the load on the foundation with respect to the **vertical**

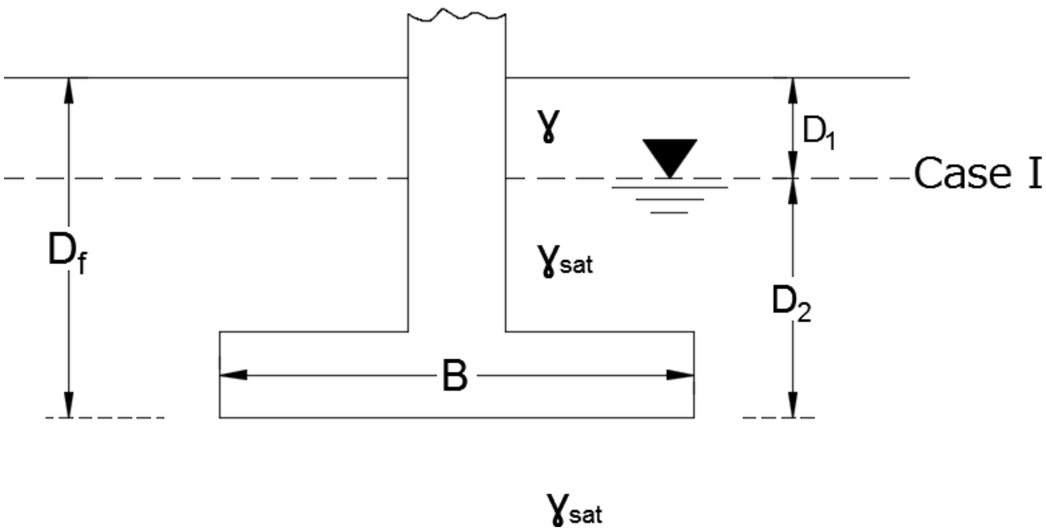
ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

Modification of Bearing Capacity Equations for Water Table

The values which will be modified are:

1.(q for soil above the foundation) in the second term of equations.

2.(γ for the underlying soil) in the third (last) term of equations .

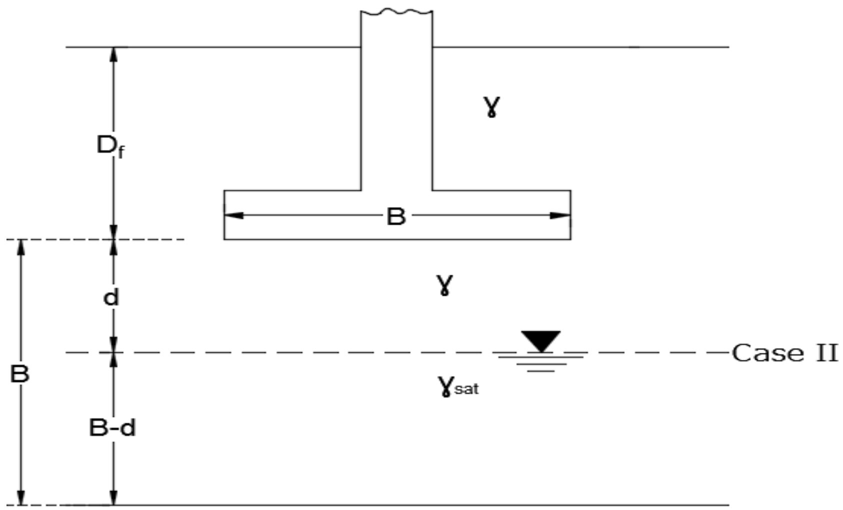


Case I. The water table is located so that $0 \le D_1 \le D_f$

$$q = D_1 \times \gamma + D_2 \times (\gamma_{sat} - \gamma_w)$$

(For the soil under the foundation) $\gamma' = \gamma_{sat} - \gamma_w$

Case II. The water table is located so that $0 \le d \le B$



The factor ,q, (second term) $q = D_f \times \gamma$

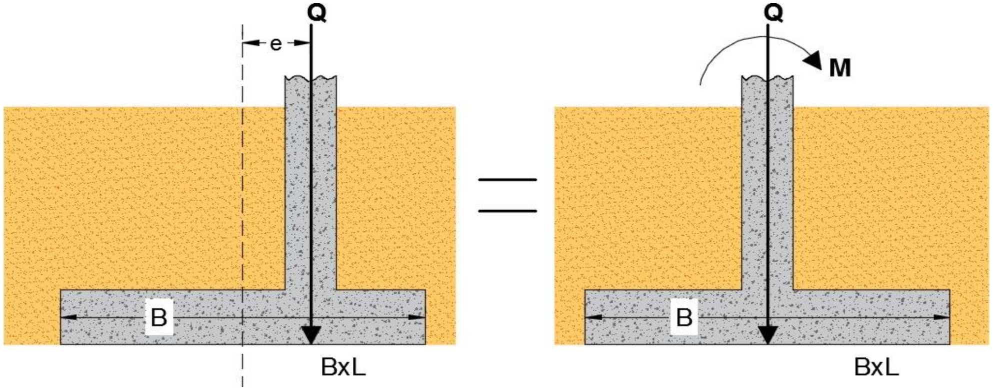
The factor , γ, (third term) $\bar{\gamma} = \gamma' + \dfrac{d \times (\gamma - \gamma')}{B}$

Case III. The water table is located so that $d \ge B$

in this case the water table is assumed have no effect on the ultimate bearing capacity

Ultimate Bearing Capacity under Eccentric Loading—One-Way Eccentricity

Eccentrically Loaded Foundation



Case I. (For $e < \frac{B}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{B}\right)$$

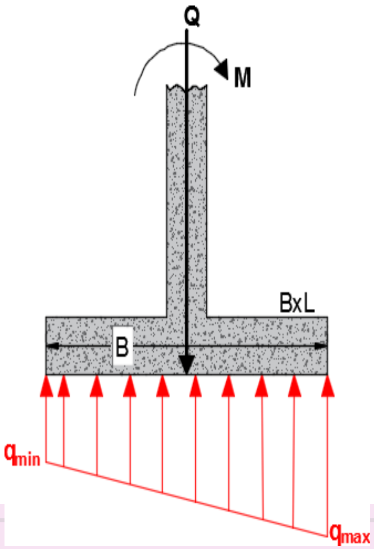
$$q_{min} = \frac{Q}{B \times L}\left(1 - \frac{6e}{B}\right)$$

If eccentricity in (L) direction:

(For $e < \frac{L}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$$q_{min} = \frac{Q}{B \times L}\left(1 - \frac{6e}{L}\right)$$



Case II. (For $e = \frac{B}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{B}\right)$$

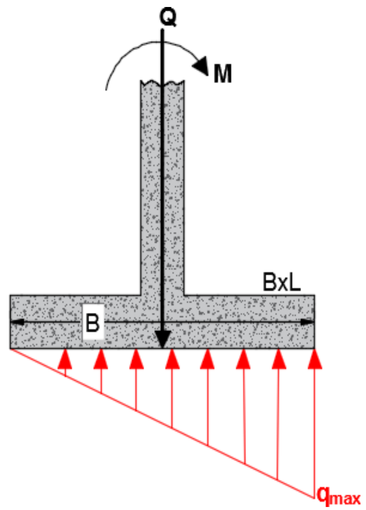
$$q_{min} = \frac{Q}{B \times L}(1 - 1) = 0.0$$

If eccentricity in (L) direction:

(For $e = \frac{L}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$$q_{min} = \frac{Q}{B \times L}(1 - 1) = 0.0$$



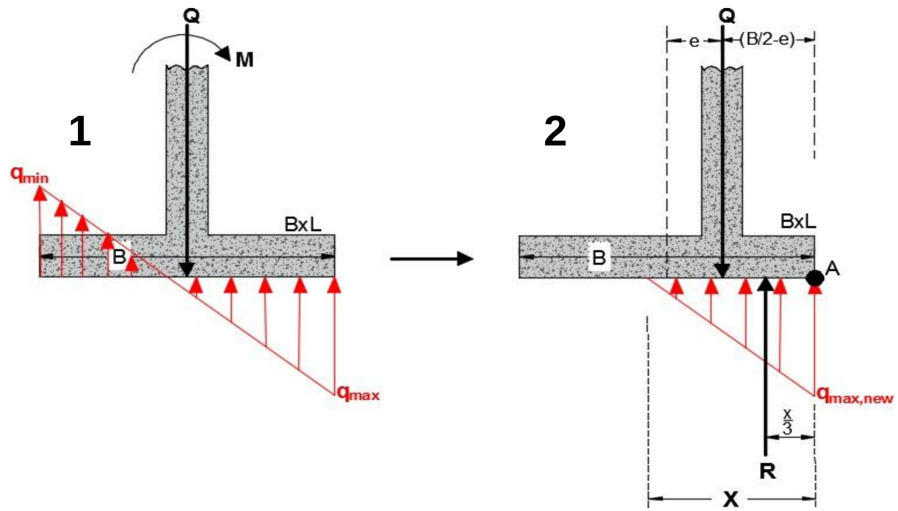
Case III. (For $e > \frac{B}{6}$):

$$q_{max,new} = \frac{4Q}{3L(B - 2e)}$$

If eccentricity in (L) direction:

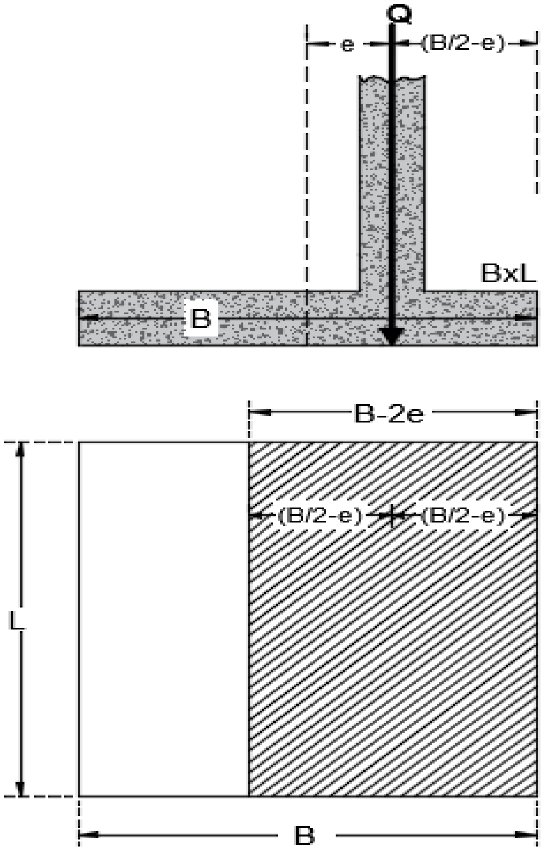
(For $e > \frac{L}{6}$):

$$q_{max,new} = \frac{4Q}{3B(L - 2e)}$$



Ultimate Bearing Capacity under Eccentric Loading—One-Way Eccentricity

Effective Area Method:



1. Determine the effective dimensions of the foundation:
effective width = B′ = B − 2e
effective Length = L′ = L

If the eccentricity were in the direction of (L) of the foundation:
Effective width = B′ = B Effective Length = L′ = L − 2e

$$B'_{used} = \min(B', L')$$
$$L'_{used} = \max(B', L')$$

2. If we want to use terzaghi's equation The value of B (in last term) will be B′

3. If we want to use Meyerhof Equation The value of B (in last term) will be B′
In calculating of shape factors (F_{cs}, F_{qs}, $F_{\gamma s}$) use B′ and L′
In calculating of depth factors (F_{cd}, F_{qd}, $F_{\gamma d}$) use the original value (B)

4. If there is a water table (Case II), we need the following equation to calculate (γ) in the last term of equations (Terzaghi and Meyerhof): $\bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B}$

The value of B used in this equation should be the original value (B)

Safety Consideration

Calculate the gross ultimate load:

$$Q_u = q_u \times \underbrace{(L'_{used} \times B'_{used})}_{A'} \qquad (A' = \text{effective area})$$

The factor of safety against bearing capacity is: $FS = \frac{Q_u}{Q_{all}} \geq 3$

Maximum Applied Load $\leq Q_{all} = \frac{Q_u}{F.S}$

The factor of safety against q_{max} is: $FS = \frac{q_u}{q_{max}} \geq 3$

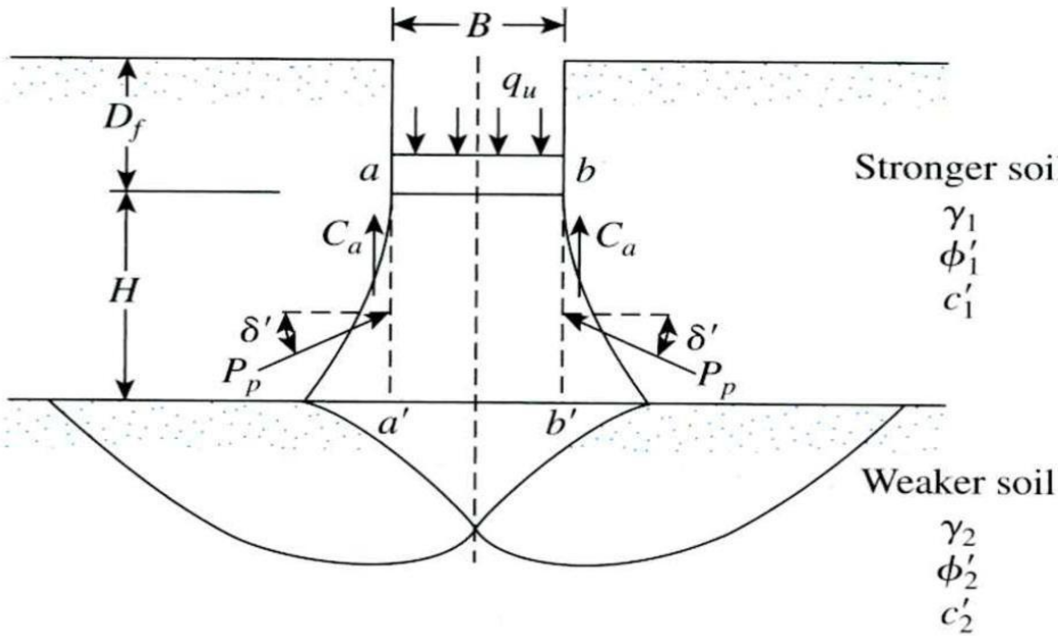
The value of q_{all} should be equal or more than q_{max}: $q_{all} \geq q_{max}$

The value of q_{min} should be equal or more than zero: $q_{min} \geq 0.0$

Bearing Capacity of Layered Soils

1. Stronger soil Underlain by Weaker Soil:

Case I: If the depth H is relatively small compared with the foundation width B (upper layer can't resist overall failure due to its small thickness), a punching shear failure will occur in the top soil layer, followed by a general shear failure in the bottom soil layer so the ultimate bearing capacity in this case will equal the ultimate bearing capacity of bottom layer (because general shear failure occur on it) in addition to punching shear resistance from top layer



$q_u = q_b$ + Punching shear resistance from top layer ($q_{punching}$)

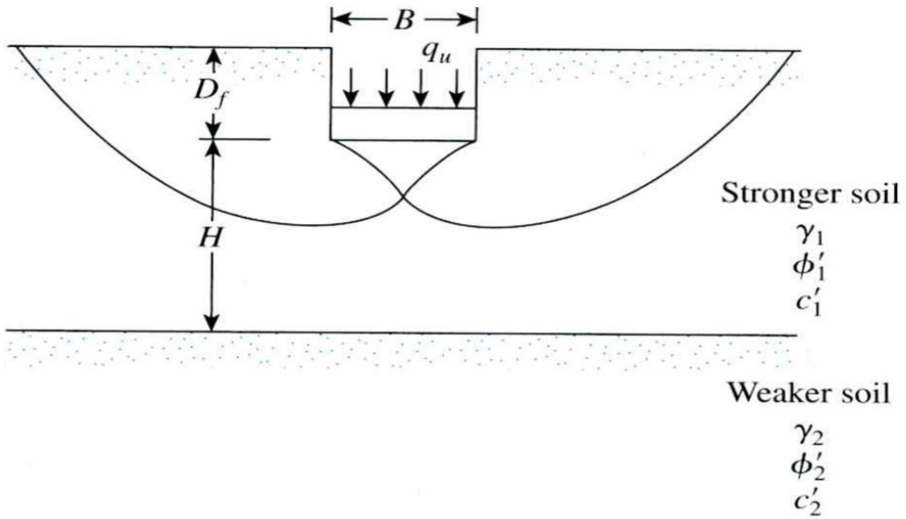
for strip or continuous footing

$$q_u = q_b + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \le q_t$$

for square, circular and rectangular footing use:

$$q_u = q_b + \left(1 + \frac{B}{L}\right) \times \frac{2c_a \times H}{B}$$
$$+\gamma_1 H^2 \times \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \le q_t$$

Case II: If the depth ,H, is relatively large (thickness off top layer is large), then the failure surface will be **completely** located in the top soil layer and the ultimate bearing capacity for this case will be the ultimate bearing capacity for top layer alone (q_t).



$$q_u = q_t = c_1 N_{c(1)} + q\, N_{q(1)} + 0.5B\gamma_1 N_{\gamma(1)}$$

$N_{c(1)}, N_{q(1)}, N_{\gamma(1)}$ = Meyerhof bearing capacity factors (for ϕ_1)

Combination of two cases:

$$q_t = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5B\gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$$

q = effective stress at the top of layer(1) = $\gamma_1 \times D_f$

$$q_b = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5B\gamma_2 N_{\gamma(2)} F_{\gamma s(2)}$$

q = effective stress at the top of layer(2) = $\gamma_1 \times (D_f + H)$

c_a = adhesion between concrete and soil along the thickness H

Ks = Punching shear coefficient

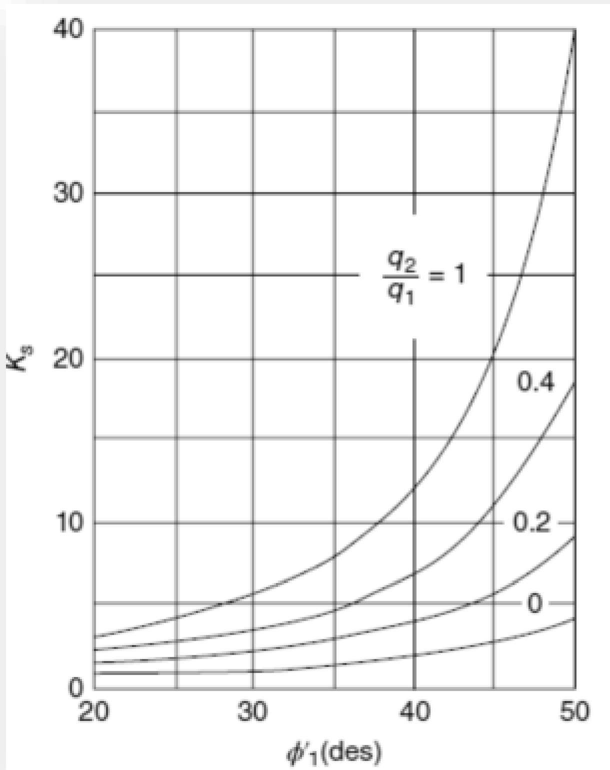
$$K_s = f\left(\frac{q_2}{q_1}, \phi_1\right) \text{ and } \frac{c_a}{c_1} = f\left(\frac{q_2}{q_1}\right)$$

Calculating of q_1 and q_2 is based on the following three main assumptions:

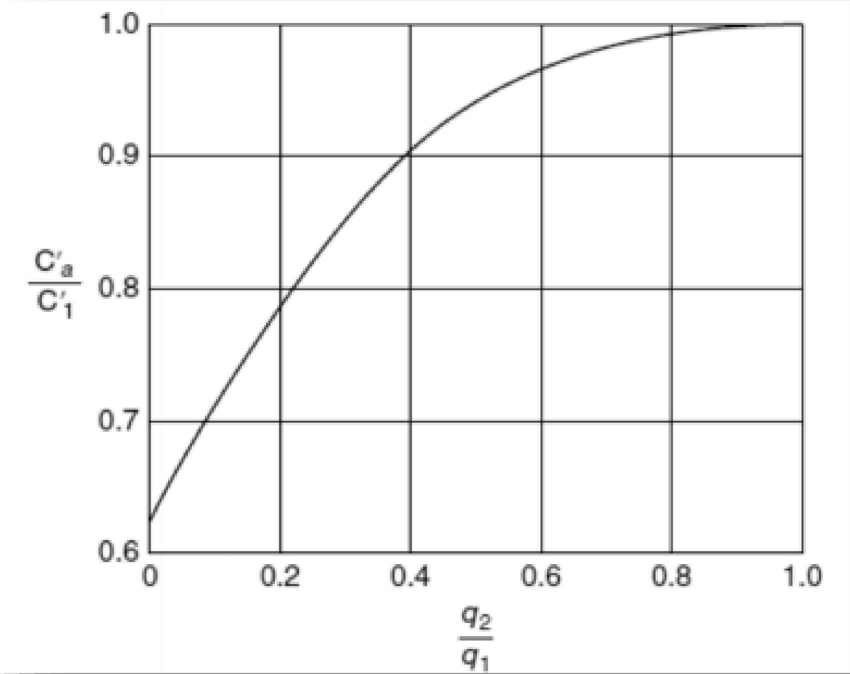
1. The foundation is always strip foundation even if it's not strip
2. The foundation exists on the ground surface ($D_f = 0.0$) and the second term on equation will be terminated.
3. In calculating q_1 we assume the top layer only exists below the foundation to a great depth, and the same in calculating of q_2.

$$q_1 = c_1 N_{c(1)} + 0.5B\gamma_1 N_{\gamma(1)}$$

$$q_2 = c_2 N_{c(2)} + 0.5B\gamma_2 N_{\gamma(2)}$$

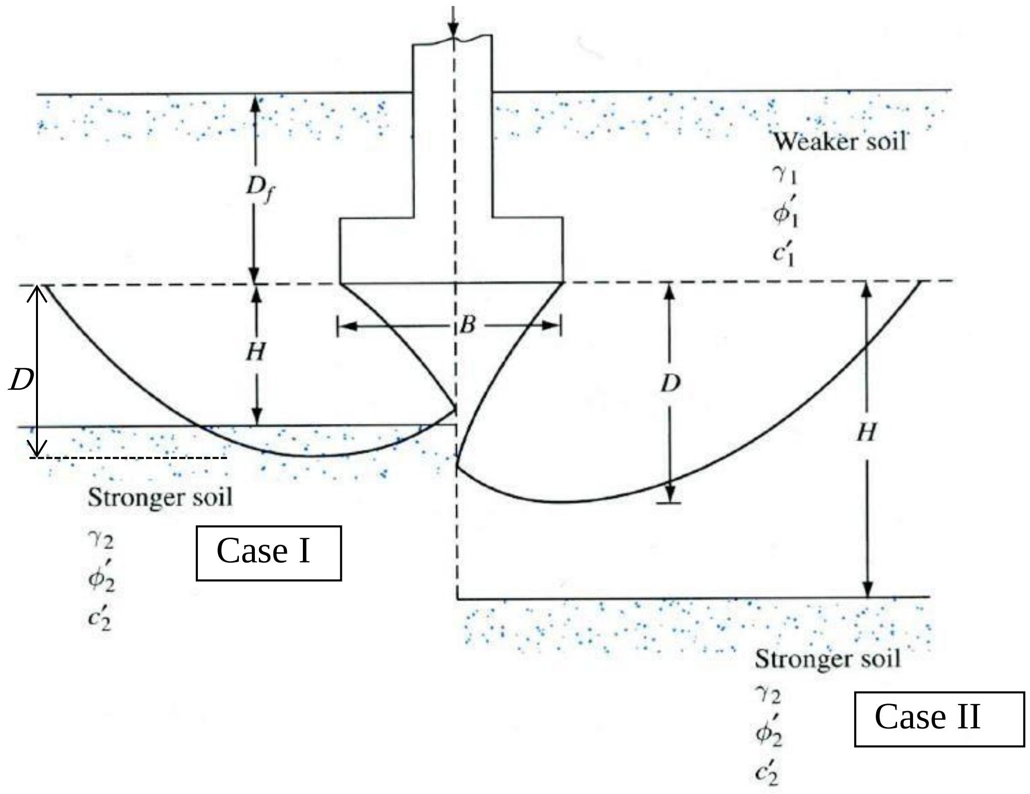


40
30
20
10
0
K_s
$\frac{q_2}{q_1} = 1$
0.4
0.2
0
20
30
40
50
ϕ'_1(des)



1.0
0.9
0.8
0.7
0.6
$\frac{C'_a}{C'_1}$
0
0.2
0.4
0.6
0.8
1.0
$\frac{q_2}{q_1}$

2.Weaker soil Underlain by Stronger Soil



Case I: $(H < D \rightarrow \frac{H}{D} < 1)$ The failure surface in soil at ultimate load will pass through both soil layers (i.e. the ultimate bearing capacity of soil will be greater than the ultimate bearing capacity for bottom layer alone).

$$\text{For } (H \leq D \rightarrow \frac{H}{D} \leq 1) \Longrightarrow q_u = q_t + (q_b - q_t)\left(1 - \frac{H}{D}\right)^2$$

Case II: $(H > D \rightarrow \frac{H}{D} > 1)$ The failure surface on soil will be fully located on top ,weaker soil layer, (i.e. the ultimate bearing capacity in this case is equal the ultimate bearing capacity for top layer alone).

$$\text{For } (H > D \rightarrow \frac{H}{D} > 1) \Longrightarrow q_u = q_t$$

$$q_{t,weak} = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5 B \gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$$

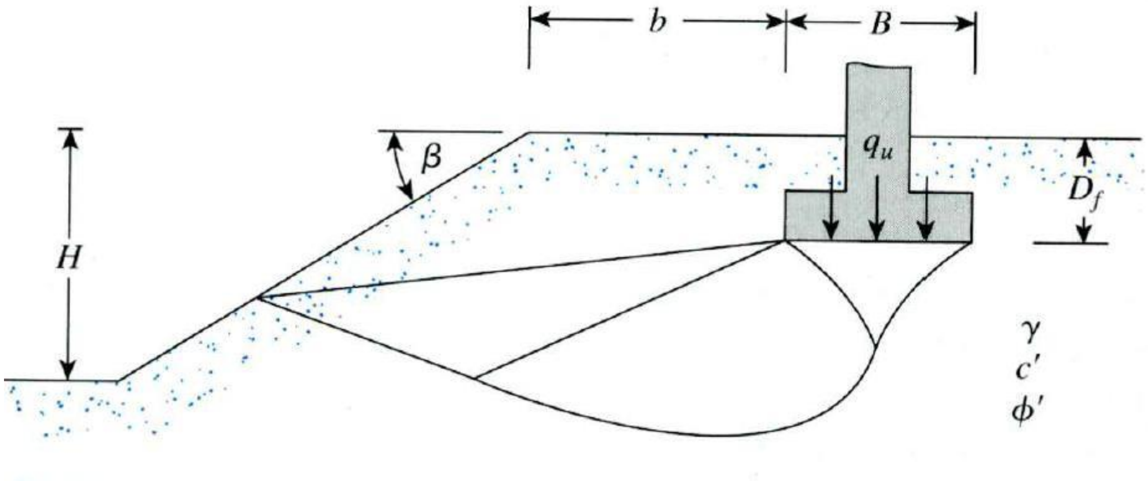
$$q_{b,strong} = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5 B \gamma_1 N_{\gamma(2)} F_{\gamma s(2)}$$

Important Note:

$D = B$ (for **loose sand** and **clay**)

$D = 2B$ (for **dense sand**)

Bearing Capacity of Foundations on Top of a Slope



H = height of slope

β = angle between the slope and horizontal

b = distance from the edge of the foundation to the top of the slope

$$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$$

For purely granular soil ($c = 0.0$):

$q_u = 0.5B\gamma N_{\gamma q}$

For purely cohesive soil ($\phi = 0.0$):

$q_u = cN_{cq}$

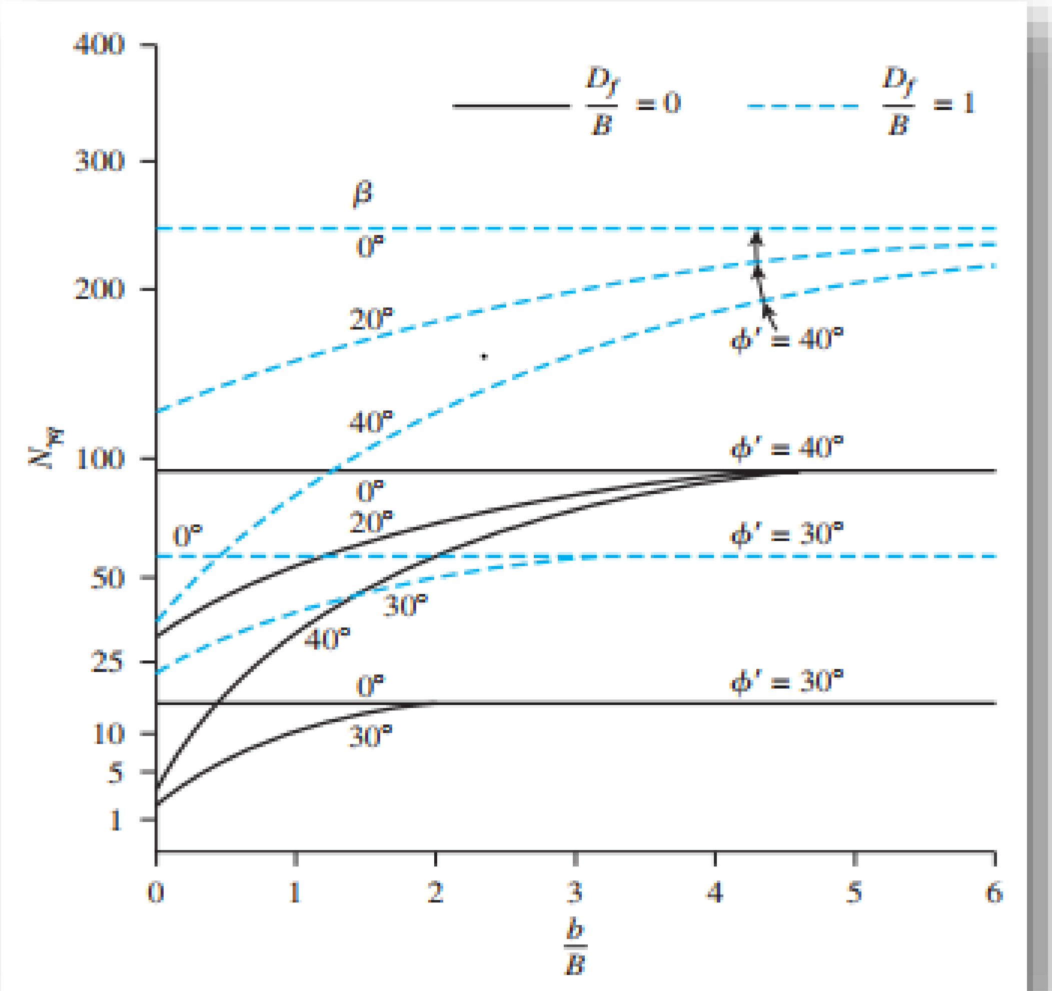
Calculating of $N_{\gamma q}$:

1. Calculate the value of $\left(\frac{D_f}{B}\right)$.
2. If $\left(\frac{D_f}{B}\right) = 0.0 \rightarrow$ use **solid lines** on the figure.
3. If $\left(\frac{D_f}{B}\right) = 1 \rightarrow$ use **dashed lines** on the figure.
4. Calculate the value of $\left(\frac{b}{B}\right)$ which the horizontal axis aof the figure.
5. According the values of (ϕ, β and factors mentioned above) we can calculate the value of $N_{\gamma q}$ on vertical axis of the figure.

Note:

If the value of $\frac{D_f}{B}$ is in the following range: $\left(0 < \frac{D_f}{B} < 1\right)$ do the following:

- ✓ Calculate $N_{\gamma q}$ at $\left(\frac{D_f}{B}\right) = 1$.
- ✓ Calculate $N_{\gamma q}$ at $\left(\frac{D_f}{B}\right) = 0$.
- ✓ Do interpolation between the above two values of $N_{\gamma q}$ to get the required value of $N_{\gamma q}$.



Meyerhof's bearing capacity factor $N_{\gamma q}$ for granular soil ($c' = 0$)

Calculating of N_{cq}:

1. Calculate the value of $\left(\frac{D_f}{B}\right)$.

2. If $\left(\frac{D_f}{B}\right) = 0.0 \rightarrow$ use **solid lines** on the figure.

3. If $\left(\frac{D_f}{B}\right) = 1 \rightarrow$ use **dashed lines** on the figure.

4. Determining the horizontal axis of the figure:

   ✓ If $B < H \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{B}\right)$

   ✓ If $B \geq H \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{H}\right)$

5. Calculating the value of stability number for clay (N_s):

   ✓ If $B < H \rightarrow$ use $N_s = 0.0$ in the figure

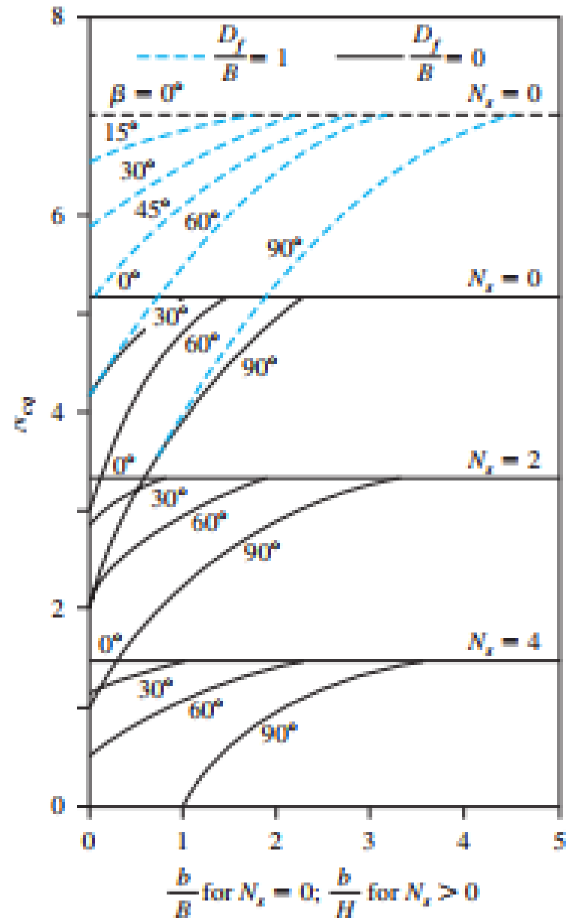
   ✓ If $B \geq H \rightarrow$ calculate N_s from this relation $N_s = \frac{\gamma H}{c}$ to be used in the figure.

6. According the values of (ϕ, β and factors mentioned above) we can calculate the value of N_{cq} on vertical axis of the figure.

Note:

If the value of $\frac{D_f}{B}$ is in the following range: $\left(0 < \frac{D_f}{B} < 1\right) \rightarrow$

Do interpolation as mentiond above.

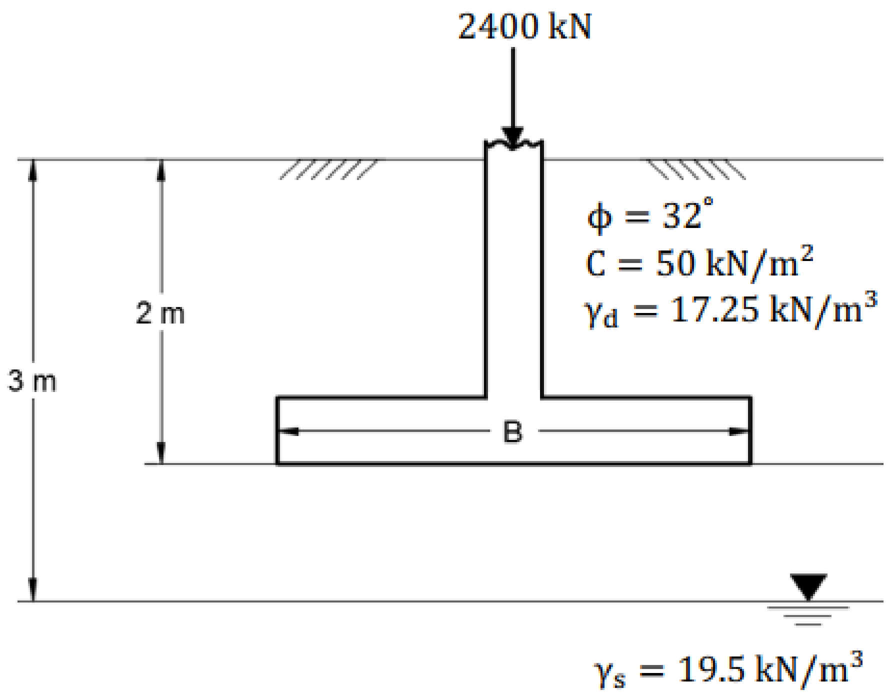


Meyerhof's bearing capacity factor N_{cq} for purely cohesive soil

Problems

1 The square footing shown below must be designed to carry a 2400 KN load. Use Terzaghi's bearing capacity formula and factor of safety = 3. **Determine** the foundation dimension B in the following two cases:

1. The water table is at 1m below the foundation (as shown).
2. The water table rises to the ground surface.



Solution

1.

$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$

$q_u = q_{all} \times FS \qquad (q_{all} = \dfrac{Q_{all}}{Area} \;,\; FS = 3)$

Applied load $\leq Q_{all} \rightarrow Q_{all} = 2400kN$

$q_{all} = \dfrac{Q_{all}}{Area} = \dfrac{2400}{B^2}$, $FS = 3 \rightarrow\rightarrow q_u = \dfrac{3 \times 2400}{B^2}$

$c = 50\ kN/m^2$

$q(\text{effective stress}) = \gamma \times D_f = 17.25 \times 2 = 34.5\ kN/m^2$

Since the width of the foundation is not known, assume $d \leq B$

$\gamma = \bar{\gamma} = \gamma' + \dfrac{d \times (\gamma - \gamma')}{B}$

$\gamma' = \gamma_{sat} - \gamma_w = 19.5 - 10 = 9.5kN/m^3 \;,\; d = 3 - 2 = 1m$

$\rightarrow \bar{\gamma} = 9.5 + \dfrac{1 \times (17.25 - 9.5)}{B} \rightarrow \bar{\gamma} = 9.5 + \dfrac{7.75}{B}$

Assume general shear failure

Note:

Always we design for general shear failure (soil have a high compaction ratio) except if we can't reach high compaction, we design for local shear

For $\phi = 32^\circ \rightarrow N_c = 44.04,\ N_q = 28.52,\ N_\gamma = 26.87$

$$\frac{7200}{B^2} = 1.3 \times 50 \times 44.04 + 34.5 \times 28.52 + 0.4 \times B \times \left(9.5 + \frac{7.75}{B}\right) \times 26.87$$

$$\frac{7200}{B^2} = 3923.837 + 102.106\,B$$

Multiply both sides by $(B^2) \rightarrow\ 102.106\,B^3 + 3923.837B^2 - 7200 = 0.0$

$\rightarrow B = 1.33m$ ✓.

2.

All factors remain unchanged except q and γ:

$q(\text{effective stress}) = (19.5 - 10) \times 2 = 19\ kN/m^2$

$\gamma = \gamma' = 19.5 - 10 = 9.5\ kN/m^3$

Substitute in terzaghi equation:

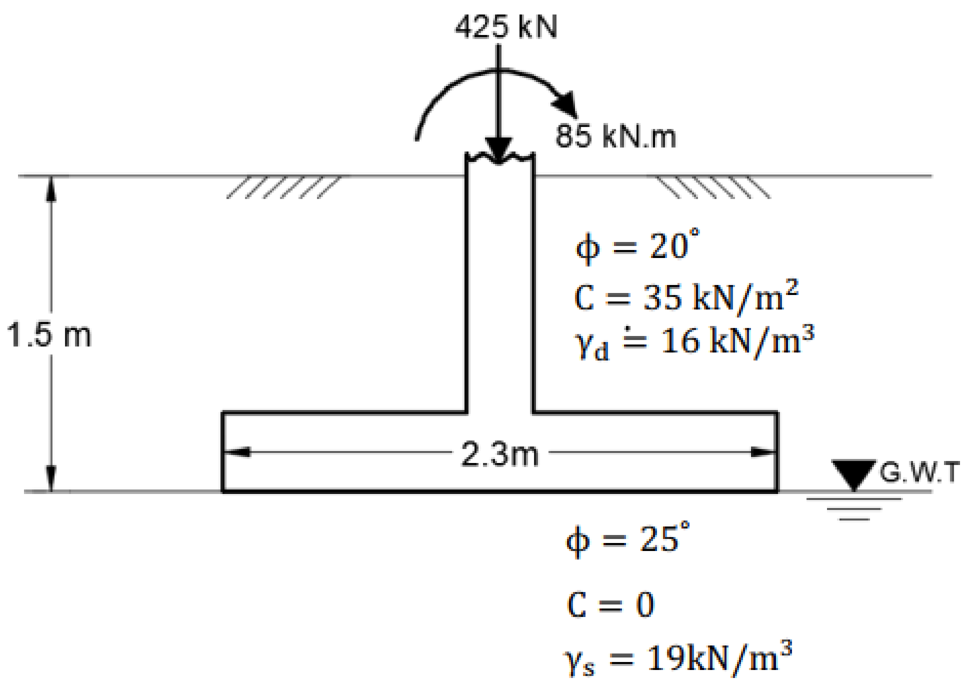
$$\frac{7200}{B^2} = 1.3 \times 50 \times 44.04 + 19 \times 28.52 + 0.4 \times B \times 9.5 \times 26.87$$

$$\frac{7200}{B^2} = 3404.48 + 102.106B$$

Multiply both sides by $(B^2) \rightarrow 102.106\,B^3 + 3404.48\,B^2 - 7200 = 0.0$

$\rightarrow B = 1.42m$ ✓.

2 For the soil profile is given below, determine the allowable bearing capacity of the isolated rectangular footing (2m x 2.3m) that subjected to a vertical load (425 kN) and moment (85 kN.m), FS=3.



Solution

$$e = \frac{M}{Q} = \frac{85}{425} = 0.2\text{m}$$

$$B' = B = 2\text{ m} \rightarrow \;,\quad L' = L - 2e \rightarrow L' = 2.3 - 2 \times 0.2 = 1.9\text{m}$$

$$B'_{used} = \min(B', L') = 1.9\text{ m} \;,\quad L'_{used} = 2\text{ m}$$

$$\text{Effective Area } (A') = 1.9 \times 2 = 3.8\text{ m}^2$$

Water table is at the bottom of the foundation $\rightarrow \gamma = \gamma' = \gamma_s - \gamma_w$

$$\rightarrow \gamma = \gamma' = 19 - 10 = 9\text{ kN/m}^3$$

For $\phi = 25^\circ \rightarrow N_c = 20.72,\; N_q = 10.66,\; N_\gamma = 10.88$

Shape Factors:

$$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right) \text{ does not required (because } c = 0.0)$$

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{1.9}{2}\right) \times \tan 25 = 1.443$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{1.9}{2}\right) = 0.62$$

Depth Factors:

$$\frac{D_f}{B} = \frac{1.5}{2} = 0.75 < 1 \quad \text{and} \quad \phi = 25 > 0.0 \rightarrow\rightarrow\rightarrow$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$ does not required (because c = 0.0)

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2 \left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan25 \times (1 - \sin25)^2 \times 0.75 = 1.233$$

$$F_{\gamma d} = 1$$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$q_u = 24 \times 10.66 \times 1.443 \times 1.233 + 0.5 \times 1.9 \times 9 \times 10.88 \times 0.62 \times 1$$

$$\rightarrow q_u = 512.87\ \text{kN/m}^2$$

$$q_{all} = \frac{q_u}{3} = \frac{512.87}{3} = 170.95\ \text{kN/m}^2 \checkmark$$

Now, we check for $q_{max} \rightarrow q_{max} \leq q_{all}$

$$\frac{L}{6} = \frac{2.3}{6} = 0.38\text{m} \rightarrow e = 0.2 < \frac{L}{6} = 0.38 \rightarrow\rightarrow$$

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

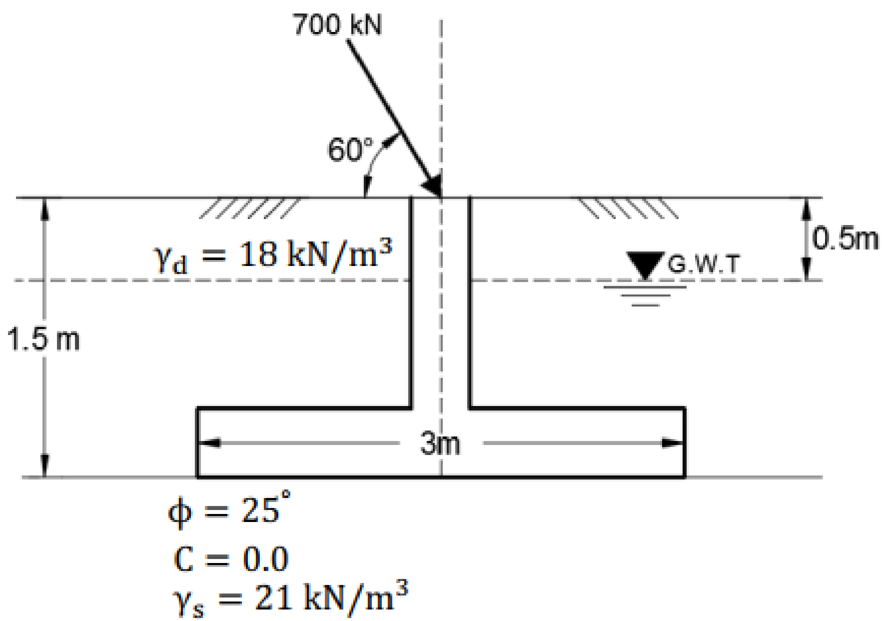
$$q_{max} = \frac{425}{2 \times 2.3}\left(1 + \frac{6 \times 0.2}{2.3}\right) = 140.6\text{kN/m}^2 < q_{all} = 170.95\ \text{kN/m}^2$$

So, the allowable bearing capacity of the foundation is $170.95\ \text{kN/m}^2$ ✓.

3 For the rectangular foundation (2m x 3m) shown below:

a) Compute the net allowable bearing capacity (FS=3).

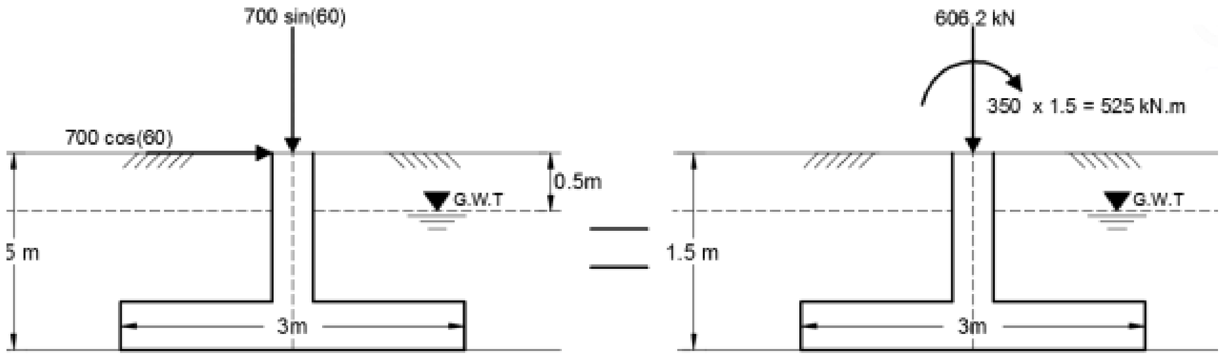
b) If the water table is lowered by 2m. What effect on bearing capacity would occur due to the water lowering?



Solution

The load on the foundation is considered inclined when this load is applied directly on the foundation, however if the load does not applied directly on the foundation (like this problem), this load is not considered inclined.

The analysis of the inclined load (700 KN) on the column will be as shown in figure below:



The inclined load on the column will be divided into two components (vertical and horizontal):

Vertical component $= 700 \times \sin 60 = 606.2$ KN

Horizontal component $= 700 \times \cos 60 = 350$ kN

The horizontal component will exerts moment on the foundation in the direction shown in figure above:

$M = 350 \times 1.5 = 525\ \text{kN.m}$

$$e = \frac{\text{Overall moment}}{\text{Vertical Load}} = \frac{525}{606.2} = 0.866\ \text{m}$$

a)

$$q_{all,net} = \frac{q_u - q}{FS}$$

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Note that the value of (c) for the soil under the foundation equal zero, so the first term in the equation will be terminated (because we calculate the bearing capacity for soil below the foundation) and the equation will be:

$$q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$$q(\text{effective stress}) = 18 \times 0.5 + (21 - 10) \times 1 = 20\ \text{kN/m}^2$$

Calculating the new area that maintains q_u uniform:

Note that the eccentricity in the direction of (L=3)

$e = 0.866\text{m}$

$B' = B = 2\ \text{m} \rightarrow\ ,\quad L' = L - 2e \rightarrow L' = 3 - 2 \times 0.866 = 1.268\text{m}$

$B'_{used} = \min(B', L') = 1.268\ \text{m}\ ,\quad L'_{used} = 2\ \text{m}$

$\text{Effective Area } (A') = 1.268 \times 2 = 2.536\ \text{m}^2$

Water table is above the bottom of the foundation $\rightarrow \gamma = \gamma' = \gamma_s - \gamma$

$\rightarrow \gamma = \gamma' = 21 - 10 = 11\ \text{kN/m}^3$

Bearing Capacity Factors:

For $\phi = 25^\circ \rightarrow N_c = 20.72,\ N_q = 10.66,\ N_\gamma = 10.88$

Shape Factors:

As we explained previously, use B'_{used} and L'_{used}

$$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right) \quad \text{does not required (because } c = 0.0\text{)}$$

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{1.268}{2}\right) \times \tan 25 = 1.296$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{1.268}{2}\right) = 0.746$$

Depth Factors:

As we explained previously, use B not B'_{used}

$$\frac{D_f}{B} = \frac{1.5}{2} = 0.75 < 1 \quad \text{and} \quad \phi = 25 > 0.0 \rightarrow\rightarrow\rightarrow$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} \quad \text{does not required (because } c = 0.0\text{)}$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 25 \times (1 - \sin 25)^2 \times 0.75 = 1.233$$

$$F_{\gamma d} = 1$$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

$$q_u = 20 \times 10.66 \times 1.296 \times 1.233 + 0.5 \times 1.268 \times 11 \times 10.88 \times 0.746$$

$$\rightarrow q_u = 397.29\ \text{kN/m}^2$$

$$q_{all,net} = \frac{q_u - q}{FS} = \frac{397.29 - 20}{3} = 125.76\ \text{kN/m}^2\ \checkmark.$$

Now, we check for $q_{max} \rightarrow q_{max} \leq q_{all}$

$$q_{all} = \frac{q_u}{FS} = \frac{397.3}{3} = 132.4\ \text{kN/m}^2$$

To calculate q_{max} we firstly should check the value of ($e = 0.866$m)

$$\frac{L}{6} = \frac{3}{6} = 0.5\text{m} \rightarrow e = 0.866 > \frac{L}{6} = 0.5 \rightarrow\rightarrow$$

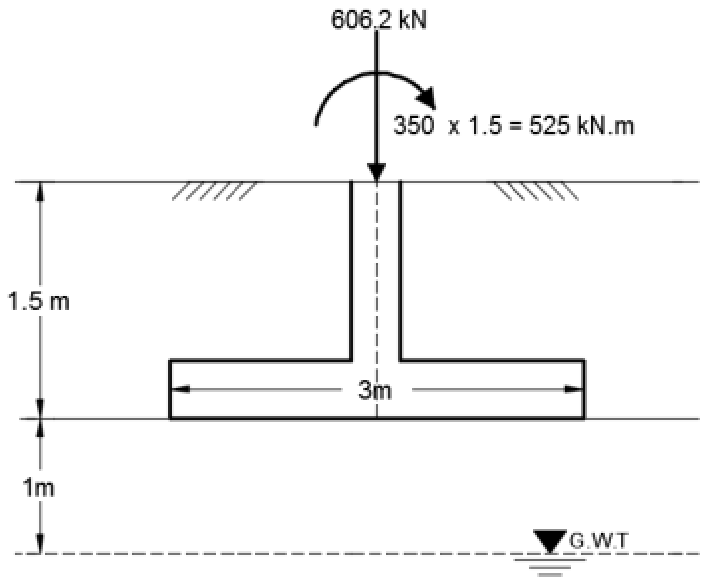
$$q_{max} = q_{max,new} = \frac{4Q}{3B(L - 2e)}$$

$$q_{max,new} = \frac{4 \times 606.2}{3 \times 2 \times (3 - 2 \times 0.866)} = 318.7\text{kN/m}^2 > q_{all} = 132.4$$

So, the allowable bearing capacity of the foundation is $132.4\ \text{kN/m}^2$ is not adequate for q_{max} and the dimensions of the footing must be enlarged.

b)

This case is shown in the below figure:



All factors remain unchanged except q and γ:

$q(\text{effective stress}) = \gamma \times D_f = 18 \times 1.5 = 27 kN/m^2$

$d = 1m \leq B = 2m \rightarrow$ water table will effect on $q_u \rightarrow\rightarrow$

$\gamma = \bar{\gamma} = \gamma' + \dfrac{d \times (\gamma - \gamma')}{B}$ (Use B not B'_{used} as we explained previously)

$\gamma' = \gamma_{sat} - \gamma_w = 21 - 10 = 11 \ kN/m^3$, $d = 1m$, $\gamma = 18 \ kN/m^3 \rightarrow$

$\bar{\gamma} = 11 + \dfrac{1 \times (18 - 11)}{2} = 14.5 \ kN/m^3$

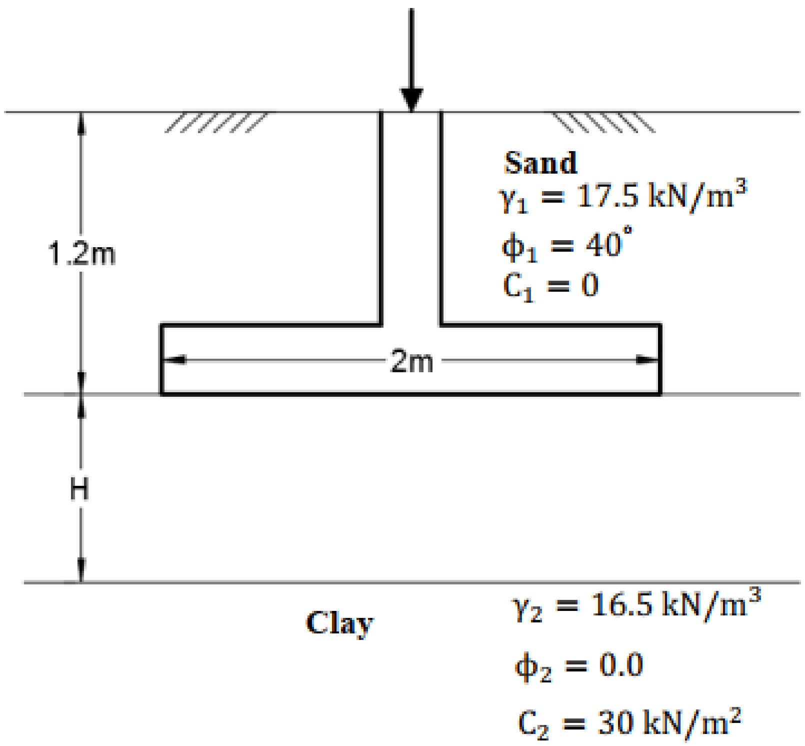
Substitute in Meyerhof equation:

$q_u = 27 \times 10.66 \times 1.296 \times 1.233 + 0.5 \times 1.268 \times 14.5 \times 10.88 \times 0.746$

$\rightarrow q_u = 534.54 \ kN/m^2$

4 The figure below shows a continuous foundation.

If H=1.5 m, determine the ultimate bearing capacity, q_u



Solution

$$q_1 = c_1 N_{c(1)} + 0.5B\gamma_1 N_{\gamma(1)} \quad (c_1 = 0.0) \rightarrow q_1 = 0.5B\gamma_1 N_{\gamma(1)}$$

$B = 2m \;,\; \gamma_1 = 17.5 \text{ kN/m}^3$

For $\phi_1 = 40^\circ \rightarrow N_{\gamma(1)} = 109.41$

$$\rightarrow q_1 = 0.5 \times 2 \times 17.5 \times 109.41 = 1914.675 \text{ kN/m}^2$$

$$q_2 = c_2 N_{c(2)} + 0.5B\gamma_2 N_{\gamma(2)} \quad (\phi_2 = 0.0) \rightarrow q_2 = c_2 N_{c(2)}$$

$c_2 = 30 \text{ kN/m}^2 \;,\;$ For $\phi_2 = 0^\circ \rightarrow N_{c(2)} = 5.14$

$$q_2 = 30 \times 5.14 = 154.2 \text{ kN/m}^2$$

$$\frac{q_2}{q_1} = \frac{154.2}{1914.675} = 0.08 < 1 \rightarrow$$ The top layer is stronger soil and bottom layer is weaker soil.

1. For strip footing:

$$q_u = q_b + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \le q_t$$

$$q_t = c_1 N_{c(1)} + q\, N_{q(1)} + 0.5B\gamma_1 N_{\gamma(1)}$$

$c_1 = 0.0 \;,\; q = \gamma_1 \times D_f = 17.5 \times 1.2 = 21 \text{kN/m}^2 \;,\; B = 2m$

For $\phi_1 = 40^\circ \rightarrow N_{c(1)} = 75.31, N_{q(1)} = 64.2, N_{\gamma(1)} = 109.41$

$$q_t = 0 + 21 \times 64.2 + 0.5 \times 2 \times 17.5 \times 109.41 = 3262.875\ kN/m^2$$

$$q_b = c_2 N_{c(2)} + q\, N_{q(2)} + 0.5B\gamma_2 N_{\gamma(2)}$$

$$c_2 = 30\ ,\ q = \gamma_1 \times (D_f + H) = 17.5 \times (1.2 + 1.5) = 47.25\ kN/m^2$$

For $\phi_2 = 0^\circ \rightarrow N_{c(2)} = 5.14, N_{q(2)} = 1, N_{\gamma(2)} = 0$

$$q_b = 30 \times 5.14 + 47.25 \times 1 + 0 = 201.45\ kN/m^2$$

Calculating of c_a:

$$\frac{q_2}{q_1} = 0.08 \rightarrow \frac{c_a}{c_1} = 0.7 \rightarrow c_a = 0.7 \times 0 = 0$$

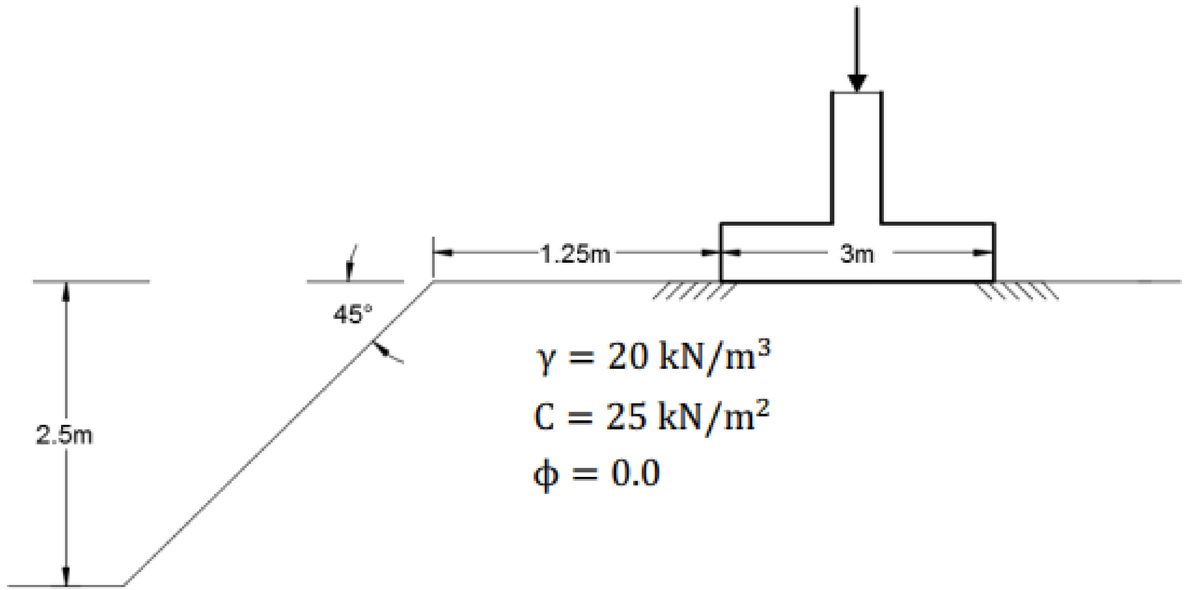
Calculating of K_s:

$$\frac{q_2}{q_1} = 0.08 \rightarrow K_s = 2.4$$

$$q_u = 201.45 + 0 + 17.5 \times 1.5^2 \left(1 + \frac{2 \times 1.2}{1.5}\right) \times \frac{2.4 \tan 40}{2} - 17.5 \times 1.5$$

$$q_u = 278\ kN/m^2 \checkmark.$$

5 For the soil profile shown below, determine the ultimate bearing capacity of the continuous footing.



$\gamma = 20\ kN/m^3$

$C = 25\ kN/m^2$

$\phi = 0.0$

Solution

From the figure: $B = 3m$, $b = 1.25\ m$, $H = 2.5m$, $D_f = 0.0m$, $\beta = 45^\circ$

$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$ but $\phi = 0.0 \rightarrow q_u = cN_{cq}$

$c = 25\ kN/m^2$

Calculating of N_{cq}

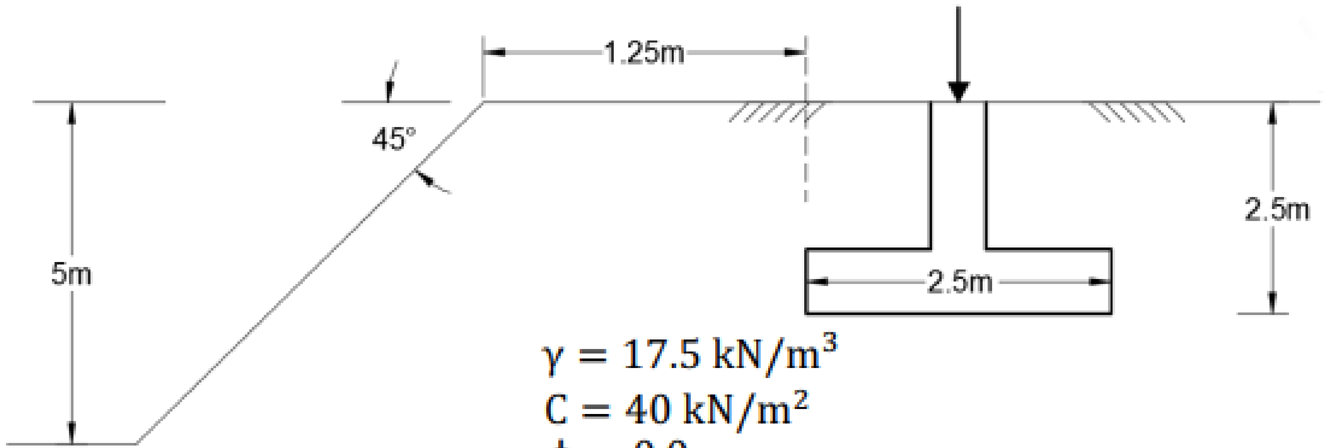
$\frac{D_f}{B} = \frac{0}{2.5} = 0 \rightarrow$ use solid lines on figure

$B = 3 > H = 2.5 \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{H}\right) = \frac{1.25}{2.5} = 0.5$

$B = 3 > H = 2.5 \rightarrow$ use $N_s = \frac{\gamma \times H}{c} = \frac{20 \times 2.5}{25} = 2$ in the figure

From the figure, the value of $N_{cq} \cong 3 \rightarrow q_u = 25 \times 3 = 75\ kN/m^2$ ✓.

6 For the soil profile shown below, determine the ultimate bearing capacity of the continuous footing.



$\gamma = 17.5\ kN/m^3$
$C = 40\ kN/m^2$
$\phi = 0.0$

Solution

From the figure: $B = 2.5m$, $b = 1.25$ m, $H = 5m$, $D_f = 2.5m$, $\beta = 45^{\circ}$

$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$ but $\phi = 0.0 \rightarrow q_u = cN_{cq}$

$c = 40\ kN/m^2$

$\frac{D_f}{B} = \frac{2.5}{2.5} = 1 \rightarrow$ use dashed lines on figure

$B = 2.5 < H = 5 \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{B}\right) = \frac{1.25}{2.5} = 0.5$

$B = 2.5 < H = 5 \rightarrow$ use $N_s = 0.0$ in the figure

From the figure, the value of $N_{cq} \cong 5.7 \rightarrow q_u = 40 \times 5.7 = 228\ kN/m^2$ ✓.