



Foundation types

Selection-design

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□ Shallow Foundations

foundations are considered to be shallow if $[D_f \leq (3 \rightarrow 4)B]$

Shallow foundations have several advantages:

- minimum cost of materials and construction
- easy in construction

the main disadvantage that if the bearing capacity of the soil supporting the foundation is small, the amount of settlement will be large.

Types of Shallow Foundations

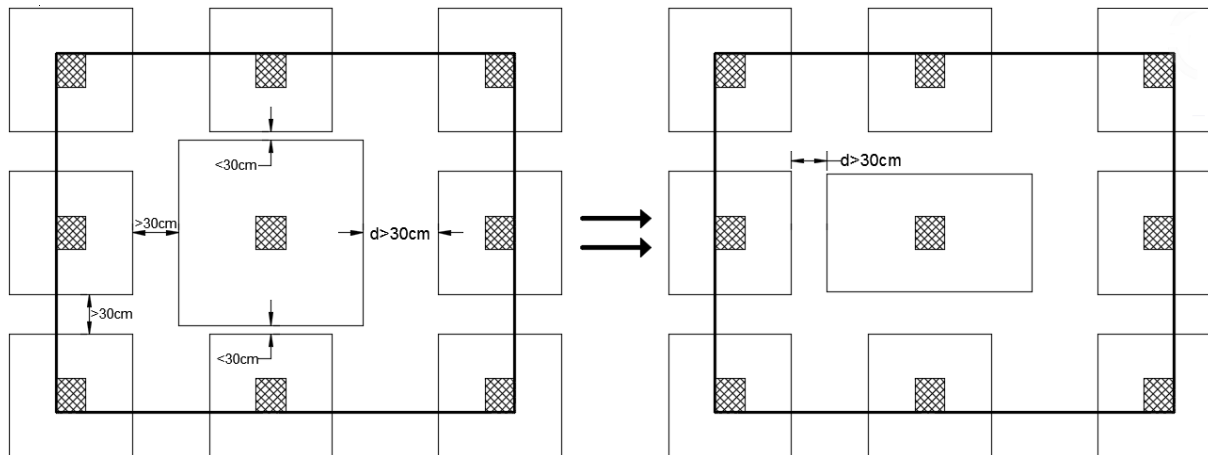
1. Isolated Footings (spread footings).
2. Combined Footings.
3. Strap Footings.
4. Mat "Raft" Foundations.



Geometric Design of Isolated Footings

The most economical type of foundations, and usually used when the loads on the columns are relatively small and the bearing capacity of the soil supporting the foundations is large.

In practice, we usually use isolated square footing because is the most economical type if the following condition is satisfied when The distance between each footing should be more than 30 cm from all direction, if not; we use isolated rectangular footing (if possible) to make the distance more than 30 cm. The following figure explains this condition:





Design:

1. Calculate the net allowable bearing capacity:

$$q_{all,net} = \frac{q_{u,net}}{FS}$$

$$q_{u,net} = q_{u,gross} - \gamma_c h_c - \gamma_s h_s$$

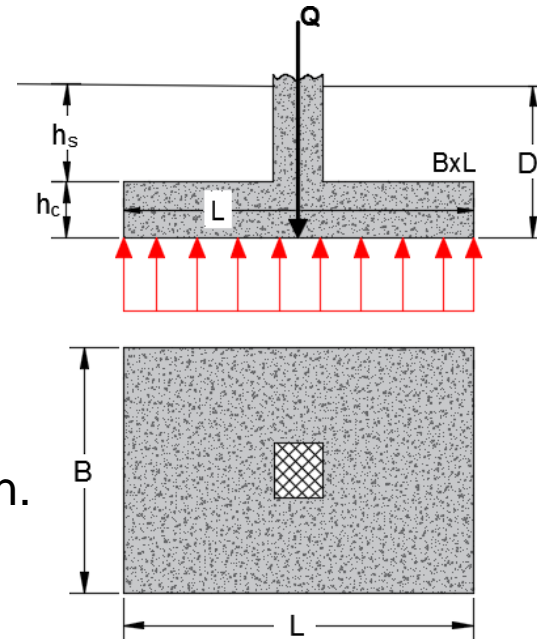
2. Calculate the required area of the footing:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L$$

Assume B or L then find the other dimension.
If the footing is square:

$$A_{req} = B^2 \rightarrow B = \sqrt{A_{req}}$$

$$Q_{service} = P_D + P_L$$





Geometric Design of Combined Footings types:

1. Rectangular Combined Footing (two columns).
2. Trapezoidal Combined Footing (two columns).
3. Strip Footing (more than two columns and may be rectangular or trapezoidal).

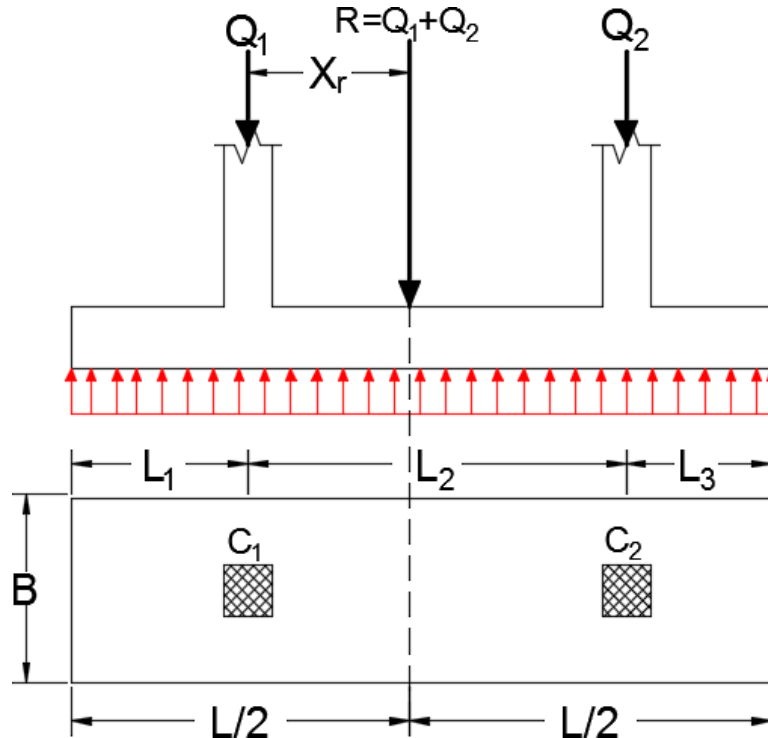
Usage:

1. Used when the loads on the columns are heavy and the distance between these columns is relatively small (i.e. when the distance between isolated footings is less than 30 cm).
2. Used as an alternative to neighbor footing which is an eccentrically loaded footing and it's danger if used when the load on the column is heavy.

Design of Rectangular Combined Footings:



1. Extension is permitted from both side of the footing:



To keep the pressure under the foundation uniform, the resultant force of all columns loads (R) must be at the center of the footing, and since the footing is rectangular, R must be at the middle of the footing (at distance $L/2$) from each edge to keep uniform pressure.



$$A_{\text{req}} = \frac{\sum Q_{\text{service}}}{q_{\text{all,net}}} = \frac{Q_1 + Q_2}{q_{\text{all,net}}} = B \times L$$

we can find L by taking summation moments

$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_2 + (W_{\text{footing}} + W_{\text{soil}}) \times X_r = R \times X_r \rightarrow X_r =$$

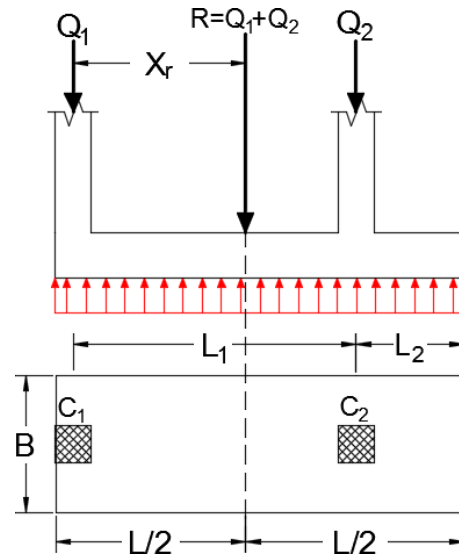
$(W_{\text{footing}} + W_{\text{soil}})$ are located at the center of the footing If we are not given any information about $(W_{\text{footing}} + W_{\text{soil}}) \rightarrow$

$$Q_2 L_2 = R \times X_r \rightarrow X_r =$$

to keep uniform pressure under the foundation:

$$X_r + L_1 = \frac{L}{2} \rightarrow L = \quad \rightarrow B = \frac{A_{\text{req}}}{L} =$$

2. Extension is permitted from one side and prevented from other side:



$$Q_1 < Q_2$$

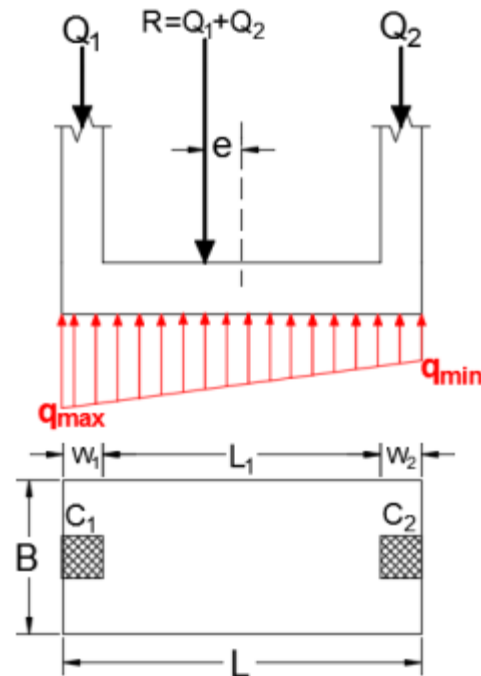
The only difference between this case and the previous case that the extension exists from one side and when we find X_r we can

easily find L:
$$X_r + \frac{\text{column width}}{2} = \frac{L}{2} \rightarrow L =$$

To keep the pressure uniform



3. Extension is not permitted from both sides of the footing:



In this case the resultant force R doesn't in the center of rectangular footing because Q_1 and Q_2 are not equals and no extensions from both sides. So the pressure under the foundation is not uniform and we design the footing in this case as following:



$$L = L_1 + W_1 + W_2 = \checkmark.$$

$$\sum M_{\text{foundation center}} = 0.0$$

$$Q_1 \times \left(\frac{L}{2} - \frac{W_1}{2}\right) - Q_2 \times \left(\frac{L}{2} - \frac{W_2}{2}\right) = R \times e$$

$$\rightarrow e = \checkmark.$$

The eccentricity in the direction of L:

Usually $e < \frac{L}{6}$ (because L is large)

$$q_{\max} = \frac{R}{B \times L} \left(1 + \frac{6e}{L}\right)$$

$q_{\text{all,gross}} \geq q_{\max} \rightarrow q_{\text{all,gross}} = q_{\max}$ (critical case)

$$q_{\text{all,gross}} = \frac{R}{B \times L} \left(1 + \frac{6e}{L}\right) \rightarrow B = \checkmark.$$

Check for B:

$$q_{\min} = \frac{R}{B \times L} \left(1 - \frac{6e}{L}\right) \text{ must be } \geq 0.0$$

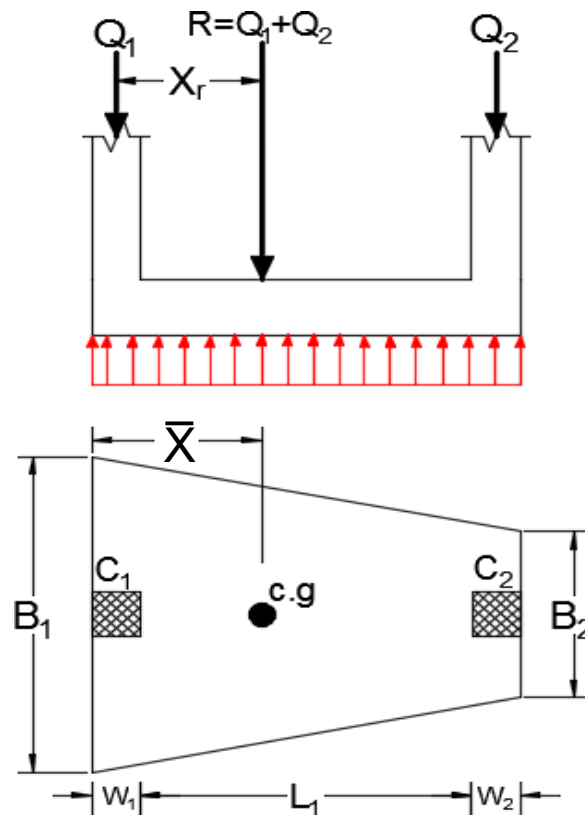
If this condition doesn't satisfied, use the modified equation for q_{\max} to find

$$q_{\max,\text{modified}} = \frac{4R}{3B(L - 2e)} \rightarrow B = \checkmark.$$



Design of Trapezoidal Combined Footings:

1. More economical than rectangular combined footing if the extension is not permitted from both sides especially if there is a large difference between columns loads.
2. We can keep uniform contact pressure in case of “extension is not permitted from both sides” if we use trapezoidal footing because the resultant force “R” can be located at the centroid of trapezoidal footing.





Design:

$Q_1 > Q_2 \rightarrow B_1 \text{ at } Q_1 \text{ and } B_2 \text{ at } Q_2$

$$L = L_1 + W_1 + W_2 = \checkmark.$$

$$A_{\text{req}} = \frac{\sum Q_{\text{service}}}{q_{\text{all,net}}} = \frac{Q_1 + Q_2}{q_{\text{all,net}}}$$

$$\frac{Q_1 + Q_2}{q_{\text{all,net}}} = \frac{L}{2} (B_1 + B_2) \rightarrow \rightarrow \text{Eq. (1)}$$

Now take summation moments at C_1 equals zero to find X_r :

$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_1 + (W_f + W_s) \times X_r = R \times X_r \rightarrow X_r = \checkmark.$$

$$X_r + \frac{W_1}{2} = \bar{X} = \checkmark.$$

$$\bar{X} = \frac{L}{3} \left(\frac{B_1 + 2B_2}{B_1 + B_2} \right) \rightarrow \rightarrow \text{Eq. (2)}$$

Solve Eq. (1) and Eq. (2) $\rightarrow \rightarrow$

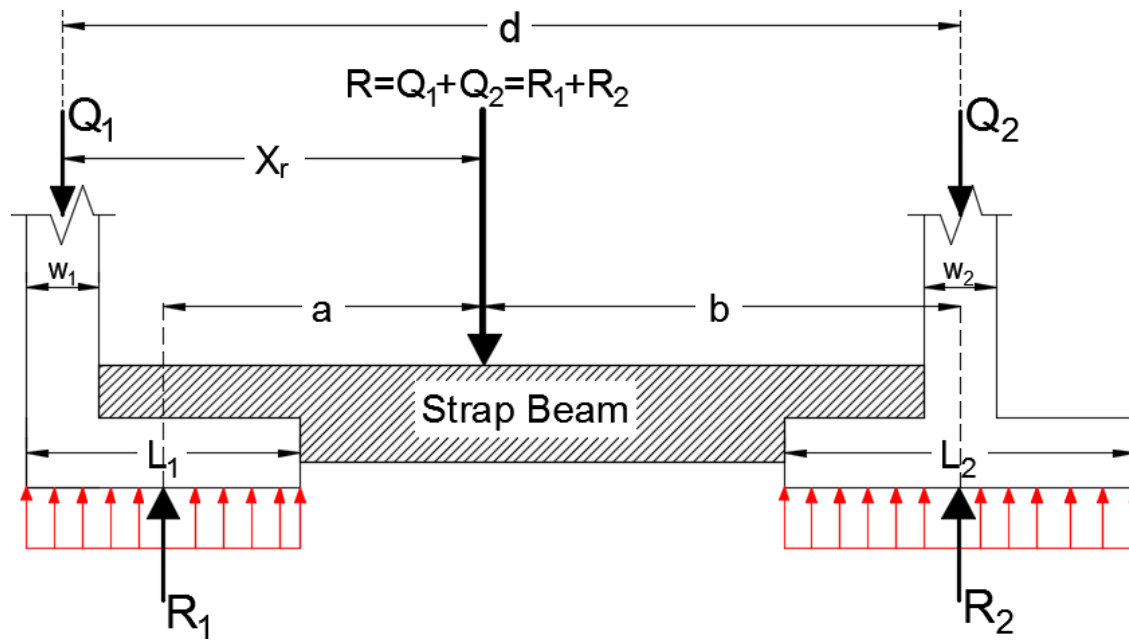
$$B_1 = \checkmark \text{ and } B_2 = \checkmark.$$

Geometric Design of Strap Footing (Cantilever Footing)



Usage:

1. Used when there is a property line which prevents the footing to be extended beyond the face of the edge column. In addition to that the edge column is relatively far from the interior column so that the rectangular and trapezoidal combined footings will be too narrow and long which increases the cost.
2. used to connect between two interior foundations one of them have a large load require a large area but this area not available, and the other foundation have a small load and there is available area to enlarge this footing, so we use strap beam to connect between these two foundations to transfer the load from largest to the smallest foundation.
3. There is a “strap beam” which connects two separated footings. The edge Footing is usually eccentrically loaded and the interior footing is centrally loaded. The purpose of the beam is to prevent overturning of the eccentrically loaded footing and to keep uniform pressure under this foundation as shown in figure below.





Design:

$R = Q_1 + Q_2 = R_1 + R_2$ but, $Q_1 \neq R_1$ and $Q_2 \neq R_2$
 Q_1 and Q_2 are knowns but R_1 and R_2 are unknowns

Finding X_r :

$$\sum M_{Q_1} = 0.0 \text{ (before use of strap beam)} \rightarrow R \times X_r = Q_2 \times d \rightarrow X_r = \checkmark$$

$$a = X_r + \frac{w_1}{2} - \frac{L_1}{2} \quad (L_1 \text{ should be assumed "if not given"})$$

$$b = d - X_r$$

Finding R_1 :

$$\sum M_{R_2} = 0.0 \text{ (after use of strap beam)} \rightarrow R_1 \times (a + b) = R \times b \rightarrow R_1 = \checkmark$$

Finding R_2 :

$$R_2 = R - R_1$$

Design:

$$A_1 = \frac{R_1}{q_{all,net}} \quad , \quad A_2 = \frac{R_2}{q_{all,net}}$$

Mat Foundation

Usage:

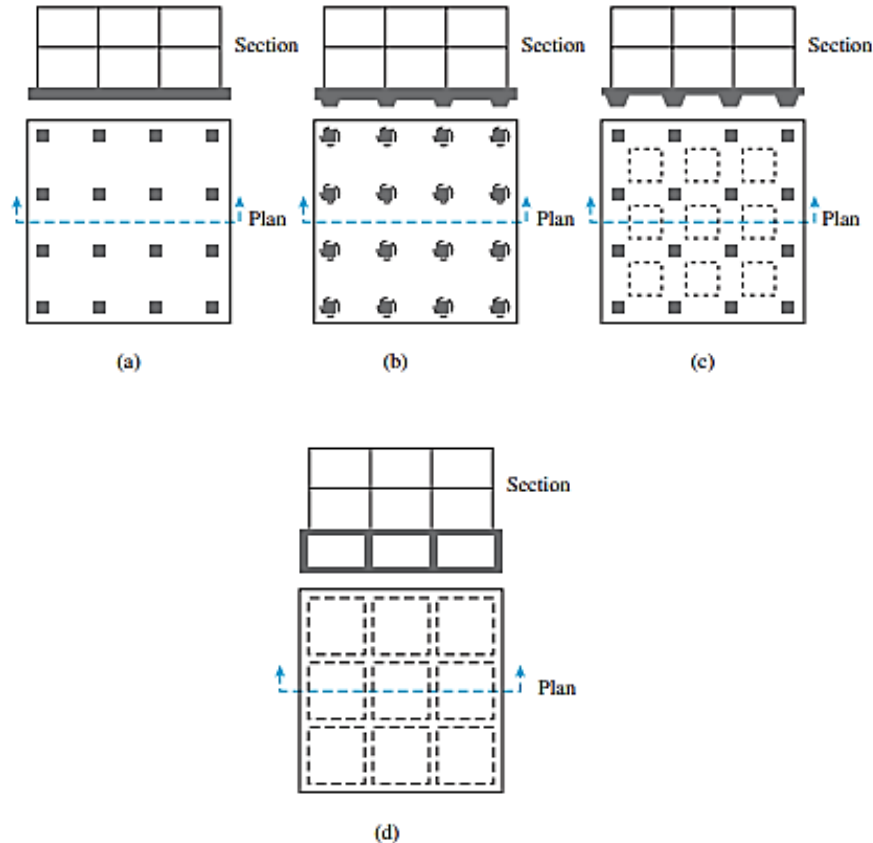
We use mat foundation in the following cases:

- 1.If the area of isolated and combined footing $> 50\%$ of the structure area, because this means the loads are very large and the bearing capacity of the soil is relatively small.
- 2.If the bearing capacity of the soil is small (usually $< 15 \text{ t/m}^2$).
- 3.If the soil supporting the structure classified as (bad soils) such as:
 - **Expansive Soil:** Expansive soils are characterized by clayey material that shrinks and swells as it dries or becomes wet respectively. It is recognized from high values of Plasticity Index, Plastic Limit and Shrinkage Limit.
 - **Compressible soil:** It contains a high content of organic material and not exposed to great pressure during its geological history, so it will be exposed to a significant settlement, so mat foundation is used to avoid differential settlement.
 - **Collapsible soil:** Collapsible soils are those that appear to be strong and stable in their natural (dry) state, but which rapidly consolidate under wetting, generating large and often unexpected settlements. This can yield disastrous consequences for structures unwittingly built on such deposits



Types:

- Flat Plate (uniform thickness). (a)
- Flat plate thickened under columns.(b)
- Beams and slabs. (c)
- Slabs with basement walls as a part of the mat. (d) .the walls act as stiffeners for the mat





Compensated Footing

The net allowable pressure **applied** on the **mat foundation** may be expressed as:

$$q = \frac{Q}{A} - \gamma D_f$$

Q = Service Loads of columns on the mat + own weight of mat

A = Area of the mat "raft" foundation.

In all cases q should be less than $q_{all,net}$ of the soil.

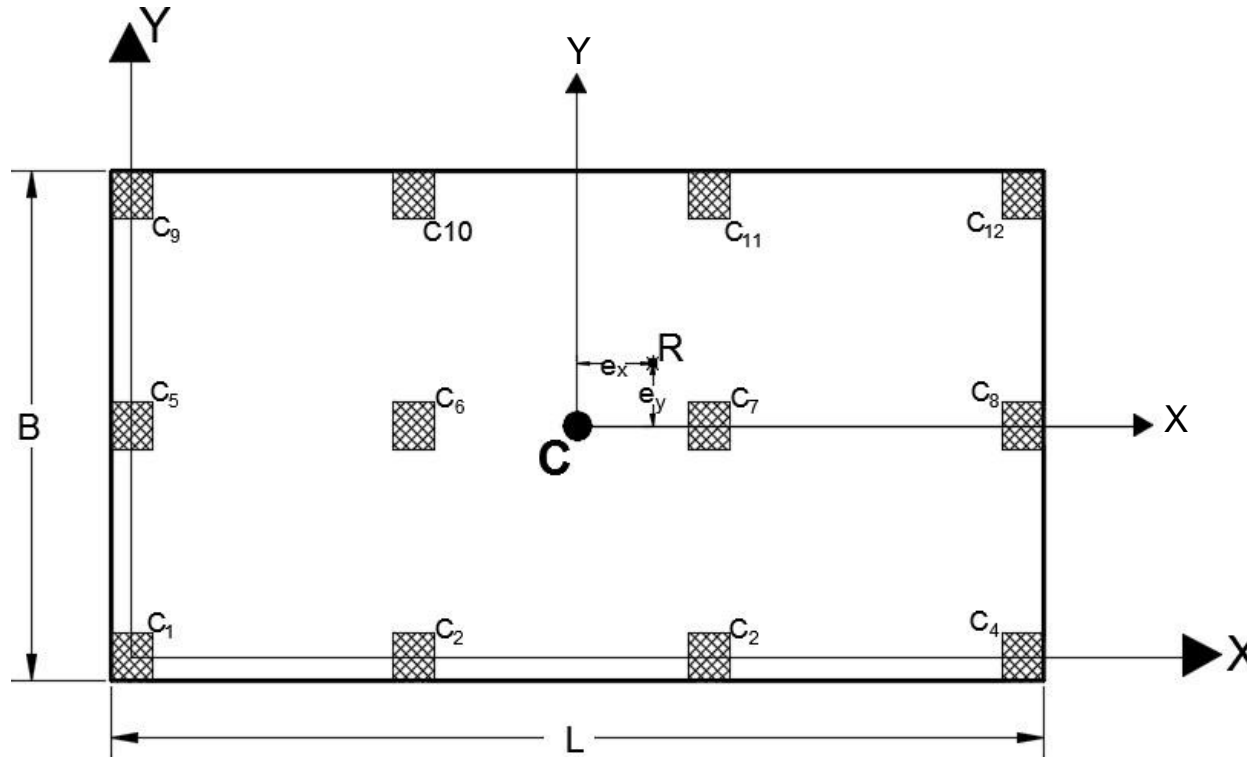
The value of q can be reduced by increasing the depth D_f of the mat, this called **compensated footing** design (i.e. replacing (substituting) the weight of the soil by the weight of the building) and is extremely useful when the when structures are to be built on very soft clay.

At the point which $q = 0.0 \rightarrow$ the overall weight of the soil above this point will replaced (substituted) by the weight of the structure and this case is called **fully compensated footing** (fully safe).

The relationship for **fully compensated depth** D_f can be determined as following:

$$0.0 = \frac{Q}{A} - \gamma D_f \rightarrow D_f = \frac{Q}{A\gamma} \text{ (fully compensated depth)}$$

Geometric Design of Mat Foundation (Working Loads)





Design:

1. Determine the horizontal and vertical axes (usually at the center line of the horizontal and vertical edge columns) as shown.

2. Calculate the centroid of the mat [point C (\bar{X} , \bar{Y})]with respect to X and Y axes:

$$\bar{X} = \frac{\sum X_i \times A_i}{\sum A_i} \quad \bar{Y} = \frac{\sum Y_i \times A_i}{\sum A_i}$$

A_i = shapes areas.

X_i = distance between y – axis and the center of the shape.

Y_i = distance between y – axis and the center of the shape.

If the mat is rectangular:

$$\bar{X} = \frac{L}{2} - \frac{w_{\text{vertical edge columns}}}{2}$$

$$\bar{Y} = \frac{B}{2} - \frac{w_{\text{horizontal edge columns}}}{2}$$

3. Calculate the resultant force R:

$$R = \sum Q_i$$



4. Calculate the location of resultant force R (X_R, Y_R) with respect to X and Y axes:

To find X_R take summation moments about Y -axis:

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$

To find Y_R take summation moments about X -axis:

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$

Q_i = load on column

X_{ri} = distance between columns center and Y – axis

Y_{ri} = distance between columns center and X – axis

5. Calculate the eccentricities:

$$e_x = |X_R - \bar{X}| \quad e_y = |Y_R - \bar{Y}|$$

6. Calculate moments in X and Y directions:

$$M_x = e_y \times \sum Q_i \quad M_y = e_x \times \sum Q_i$$



7. Calculate the stress under each corner of the mat:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y$$

$I_x = I_{\bar{x}}$ = moment of inertia about centroid of mat (in x – direction)

$I_y = I_{\bar{y}}$ = moment of inertia about centroid of mat (in y – direction)

8. Check the adequacy of the dimensions of mat foundation:

Calculate q_{\max} (maximum stress among all corners of the mat)

Calculate q_{\min} (minimum stress among all corners of the mat)

$$q_{\max} \leq q_{\text{all,net}}$$

$$q_{\min} \geq 0.0$$

If one of the two conditions doesn't satisfied, increase the dimensions of the footing.

□ pile Foundations



Piles are structural members that are made of steel, concrete, or timber.

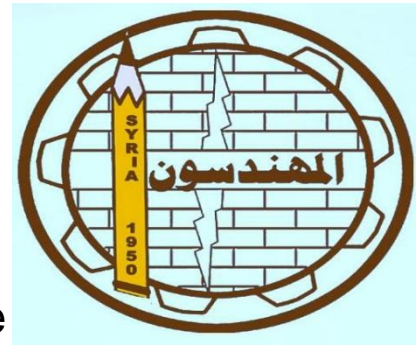
They are used to build pile foundations (classified as deep foundations) which cost more than shallow foundations.

Despite the cost, the use of piles often is necessary to ensure structural safety.

The most case in which pile foundations are required, is when the soil supporting the structure is **weak soil** (expansive soil, or collapsible soil, etc...) we use piles to transmit the foundation load to the nearest bed rock layer, and if bed rock is not encountered, we use piles to transmit the load to the nearest stronger soil layer to ensure the safety for the structure.

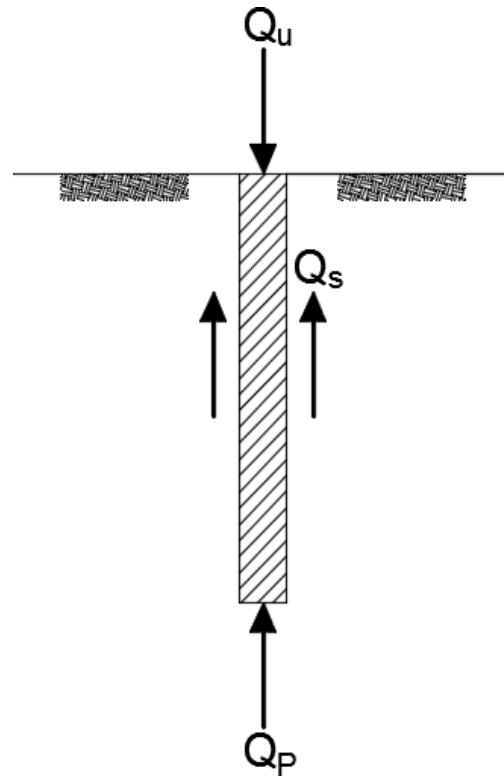
Capacity of Piles

$$Q_u = Q_P + Q_s$$



Q_P = Load carried at the pile end point

Q_s = Load carried by the skin friction developed at the sides of the pile



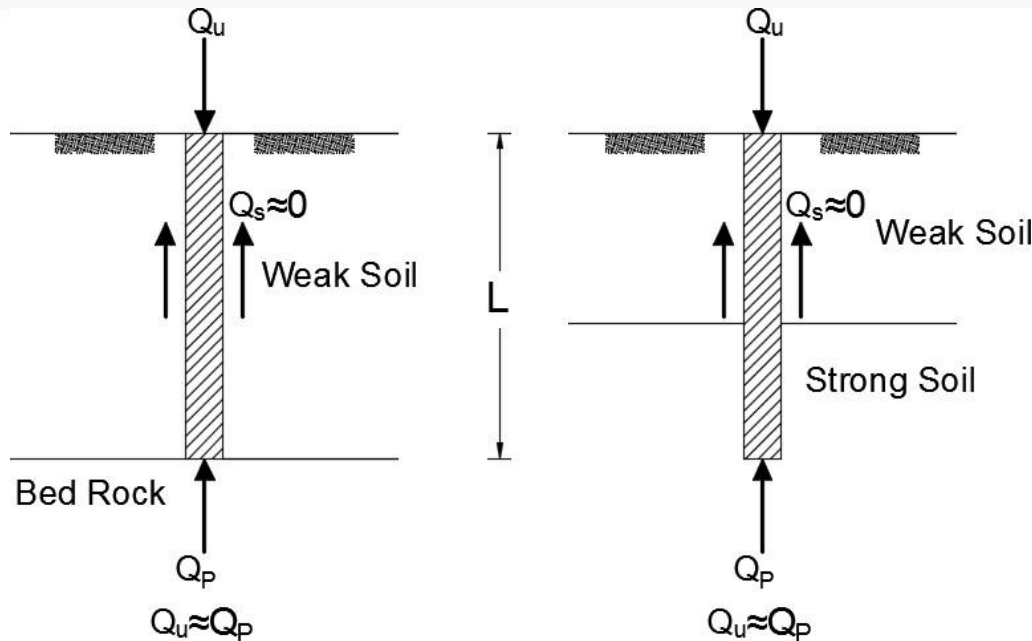


Types of Pile

1. Point Bearing Piles:

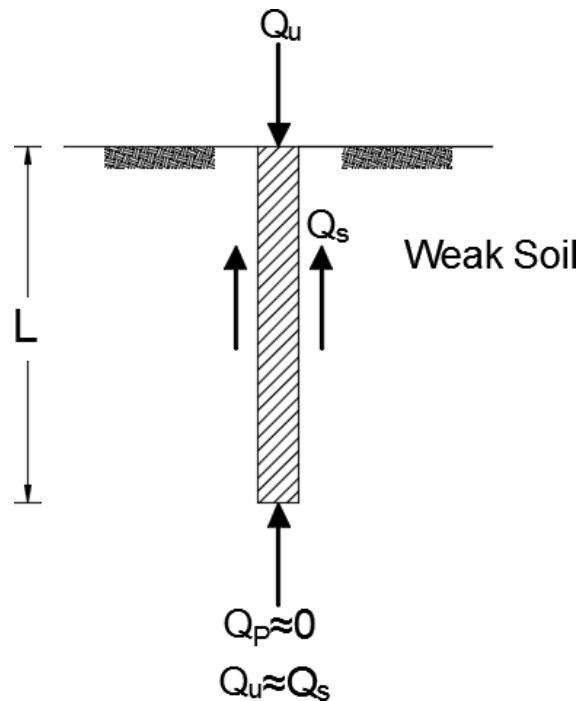
If the soil supporting the structure is weak soil, pile foundation will be used to transmit the load to the strong soil layer or to the bed rock

$$Q_u = Q_p + Q_s \quad Q_s \approx 0.0 \rightarrow Q_u = Q_p \text{ (Point Bearing Piles)}$$

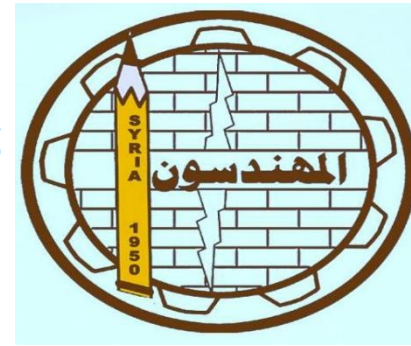


2. Friction Piles:

When no strong layer or rock is present at reasonable depth at a site, point bearing piles becomes very long (to reach strong layer) and uneconomical. In these type of soil profiles, piles are driven through the softer (weaker) soil to specified depth



$$Q_u = Q_p + Q_s \quad Q_p \approx 0.0 \rightarrow Q_u = Q_s \text{ (Friction Piles)}$$



In practice, we assume the pile resist the applied loads by its point bearing load and its frictional resistance to estimate the ultimate load the pile can carry.

$$Q_u = Q_P + Q_s$$

Calculation of Point Bearing Load(Q_P)

We will use **Meyerhof method** to calculate the value of Q_P for sand and clay.

Calculation of Q_P for sand:

$$Q_P = A_P \times q_P \leq Q_L$$

A_P = Cross-sectional area of the **end point** of the pile (**bearing area** between pile and soil).

$$q_P = q' \times N_q^*$$

$$Q_P = A_P \times q' \times N_q^* \leq 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi$$

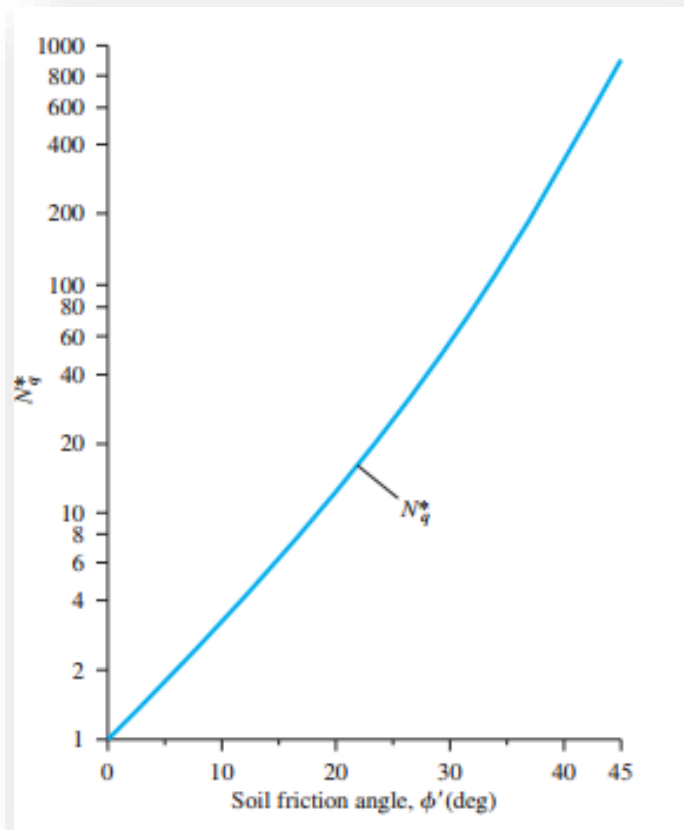
q' = **Effective vertical stress** at the level of the end of the pile.

N_q^* = Load capacity Factor (depends only on ϕ – value)



N_q^* Based on Meyerhof's Theory

Soil friction angle, ϕ (deg)	N_q^*
20	12.4
21	13.8
22	15.5
23	17.9
24	21.4
25	26.0
26	29.5
27	34.0
28	39.7
29	46.5
30	56.7
31	68.2
32	81.0
33	96.0
34	115.0
35	143.0
36	168.0
37	194.0
38	231.0
39	276.0
40	346.0
41	420.0
42	525.0
43	650.0
44	780.0
45	930.0





$$Q_P = A_P \times q' \times N_q^* \leq 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi$$

p_a = atmospheric pressure = 100 kN/m²

Calculation of Q_P for Clay:

$$Q_P = A_P \times c_u \times N_c^*$$

c_u = Cohesion for the soil supported the pile at its end.

N_c^* = Bearing capacity factor for clay = 9 (when $\phi = 0.0$)

$$Q_P = 9 \times A_P \times c_u$$

Calculation of Q_P for $C - \phi$ Soile:

$$Q_P = (A_P \times q' \times N_q^* \leq 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi) + A_P \times c_u \times N_c^*$$



Calculation of Frictional Resistance(Q_s)

Calculation of Q_s for sand:

$$Q_s = P \times \sum f_i \times L_i$$

P = pile perimeter

f_i = unit friction resistance at any depth

L_i = depth of each soil layer

$$f = \mu_s \times N$$

μ_s = friction coefficient between soil and pile = $\tan \delta$

δ = soil – pile friction angle = $0.8\phi \rightarrow \mu_s = \tan(0.8\phi)$ (for each layer)

$N = \sigma'_v \times K$ (for each soil layer)

σ'_v = vertical effective stress for each layer

K = Effective earth pressure coefficient

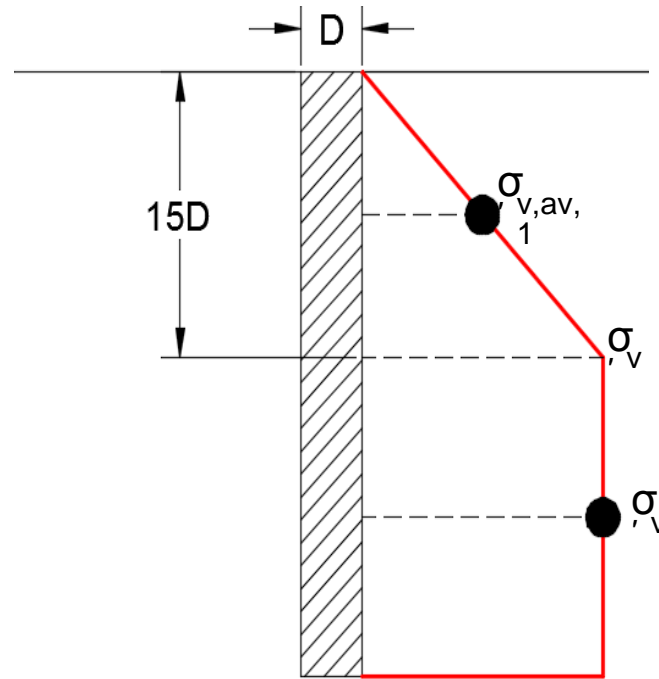
$K = 1 - \sin\phi$ or $K = 0.5 + 0.008 D_r$ D_r = relative density (%)

$N = \sigma'_{v,av} \times K$ (for each soil layer)

$f = \tan(0.8\phi) \times \sigma'_{v,av} \times K$ (for each soil layer)



calculate the value of $\sigma'_{v,av}$ for each soil layer:

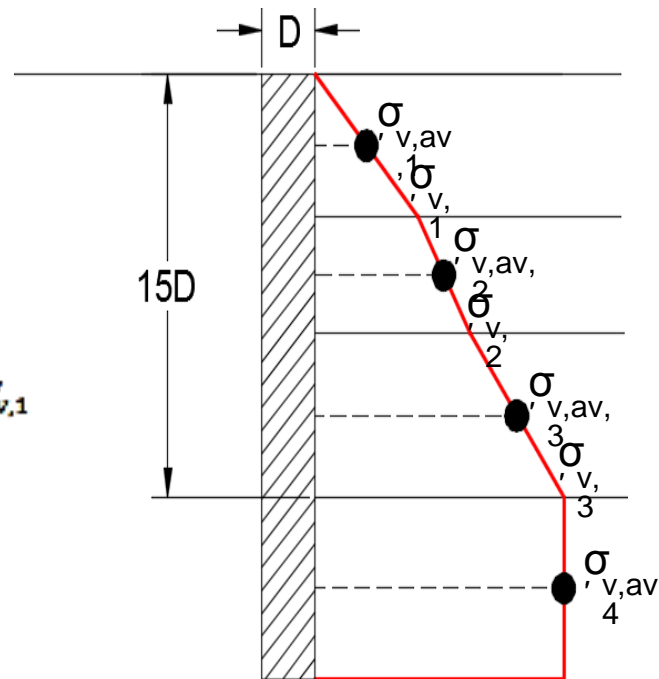


$$\sigma'_{v,av,1} = \frac{0 + \sigma'_v}{2} = 0.5\sigma'_v$$

$$\sigma'_{v,av,2} = \frac{\sigma'_v + \sigma'_v}{2} = \sigma'_v$$



If there are more than one soil layer before reaching 15D:



$$\sigma'_{v,av,1} = \frac{0 + \sigma'_{v,1}}{2} = 0.5\sigma'_{v,1}$$

$$\sigma'_{v,av,2} = \frac{\sigma'_{v,1} + \sigma'_{v,2}}{2}$$

$$\sigma'_{v,av,3} = \frac{\sigma'_{v,2} + \sigma'_{v,3}}{2}$$

$$\sigma'_{v,av,4} = \frac{\sigma'_{v,3} + \sigma'_{v,3}}{2} = \sigma'_{v,3}$$

Finally we can calculate the value of Q_s as following:

$$Q_s = P \times \sum \tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i \times L_i$$

i = each soil layer



Calculation of Q_s for clay:

There are three methods used to calculate Q_s in clay:

1. λ Method

$$Q_s = P \times \sum f_l \times L_l$$

But here we take the entire length of the pile:

$$Q_s = P \times L \times \sum f_l$$

$$\sum f_l = f_{av} = \lambda \times (\sigma'_{v,av} + 2 c_{u,av})$$

$\sigma'_{v,av}$ = mean effective vertical stress for the entire embedment length

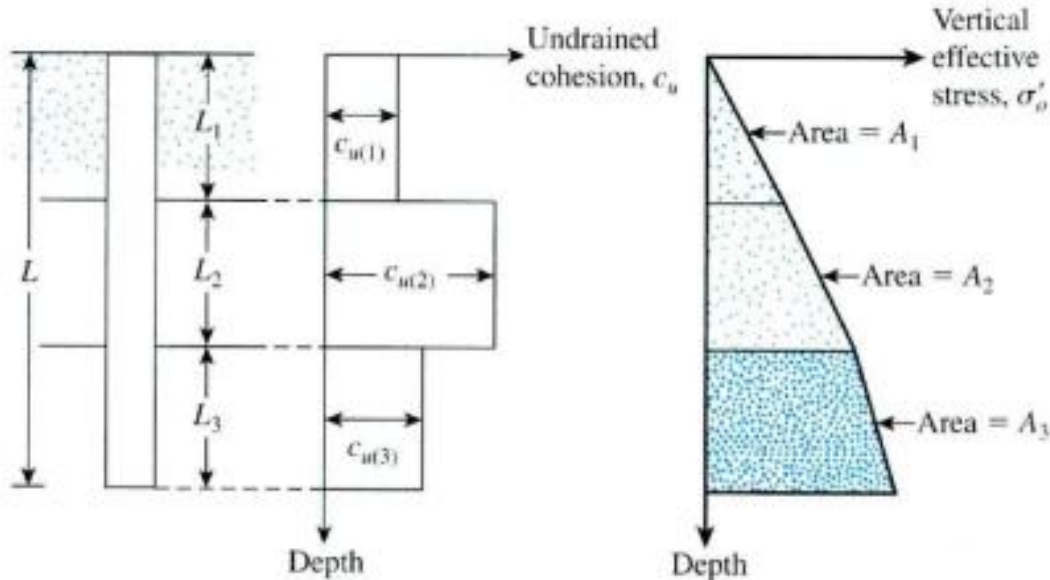
$c_{u,av}$ = mean undrained shear strength for the entire embedment length

λ = function of pile length (L)



Calculation of $\sigma'_{v,av}$ and $c_{u,av}$:

We prepare the following graph (assuming three soil layers):



Note that the soil is clay, and the stress is not constant after 15D, the stress is constant (after 15D) in sand only.

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3}{L}$$



Embedment length, L (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
60	0.113
70	0.110
80	0.110
90	0.110



2. α Method

$$Q_s = P \times \sum f_l \times L_l$$

$$f_l = \alpha_l \times c_{u,l}$$

$$\alpha_l = \text{function of } \left(\frac{c_{u,l}}{p_{atm}} \right)$$

$$Q_s = P \times \sum \alpha_l \times c_{u,l} \times L_l$$

$\frac{c_u}{p_a}$	α
≤ 0.1	1.00
0.2	0.92
0.3	0.82
0.4	0.74
0.6	0.62
0.8	0.54
1.0	0.48
1.2	0.42
1.4	0.40
1.6	0.38
1.8	0.36
2.0	0.35
2.4	0.34
2.8	0.34

Note: p_a = atmospheric pressure
 $\approx 100 \text{ kN/m}^2$



3. β Method

3. β Method

$$Q_s = P \times \sum f_l \times L_l$$

$$f_l = \beta_l \times \sigma'_{v,av,l}$$

$\sigma'_{v,av,l}$ = average vertical effective stress for each clay layer

$$\beta_l = K_l \times \tan \phi_{R,l}$$

ϕ_R = drained friction angle of remolded clay (given for each layer)

K_l = earth pressure coefficient for each clay layer

$K = 1 - \sin \phi_R$ (for normally consolidated clay)

$K = (1 - \sin \phi_R) \times \sqrt{OCR}$ (for overconsolidated clay)

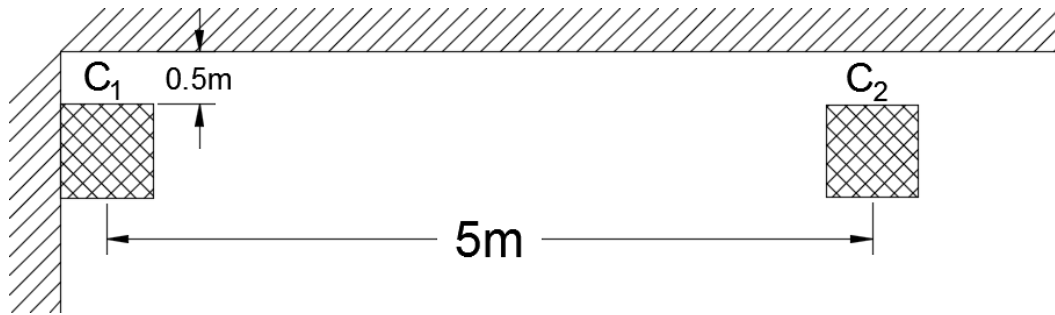
Important Note:

If the soil is (C – ϕ) soil, we calculate Q_s for sand alone and for clay alone and then sum the two values to get the total Q_s



Problems

1. Design the foundation shown below to support the following two columns with uniform contact pressure:
- Column (1): $P_D=500$ kN , $P_L=250$ kN , cross section (50cm \times 50cm).
Column (2): $P_D=700$ kN , $P_L=350$ kN , cross section (50cm \times 50cm).
Assume the net allowable soil pressure is 200 kN/m²



Solution

To keep uniform contact pressure under the base, the resultant force R must be at the center of the foundation.

Since the extension is permitted from right side, we can use rectangular combined footing.

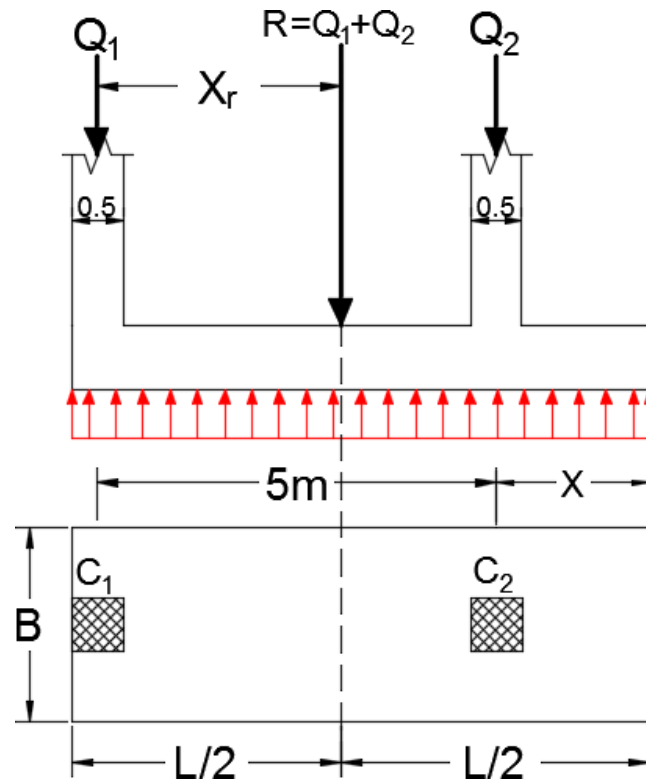
$$Q_1 = 500 + 250 = 750 \text{ kN}$$

$$Q_2 = 700 + 350 = 1050 \text{ kN}$$

$$R = Q_1 + Q_2 = 750 + 1050$$

$$R = 1800 \text{ kN}$$

The weight of the foundation and the soil is not given, so we neglect it.



$$\sum M_{C_1} = 0.0 \rightarrow 1050 \times 5 = 1800 \times X_r \rightarrow X_r = 2.92 \text{ m}$$



To keep uniform pressure: $X_r + \frac{0.5}{2} = \frac{L}{2}$

$$\frac{L}{2} = 2.92 + \frac{0.5}{2} = 3.17\text{m}$$

$$\frac{L}{2} = 3.17\text{m} \rightarrow L = 3.17 \times 2 = 6.34\text{ m} \checkmark.$$

Calculation of B:

$$A_{\text{req}} = \frac{Q_{\text{service}}}{q_{\text{all,net}}} = B \times L \rightarrow \frac{1800}{200} = B \times 6.34 \rightarrow B = 1.42\text{ m} \checkmark.$$

Check for B:

Available value for B:

The permitted extension for the width B is 0.5 m, so the available width is

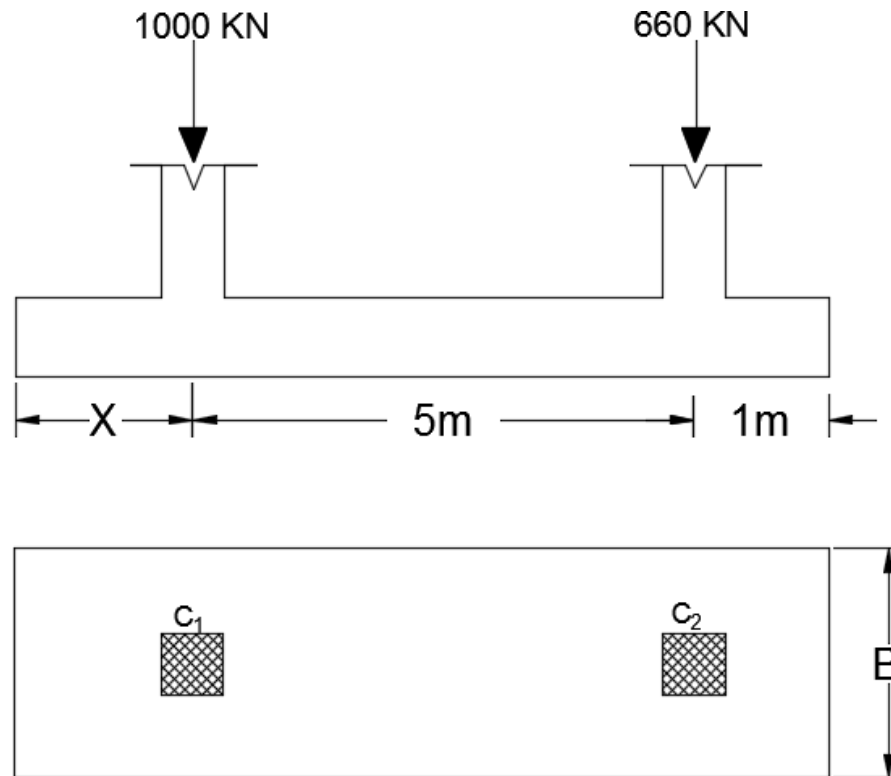
$$0.5 + (\text{column width}) + 0.5 = 0.5 + 0.5 + 0.5 = 1.5\text{ m}$$

$$B = 1.42\text{ m} < 1.5\text{m Ok}$$



2. For the combined footing shown below.

- Find distance X so that the contact pressure is uniform.
- If $q_{all,net} = 140 \text{ kN/m}^2$, find B .





Solution

To keep uniform contact pressure under the base, the resultant force R must be at the center of the foundation.

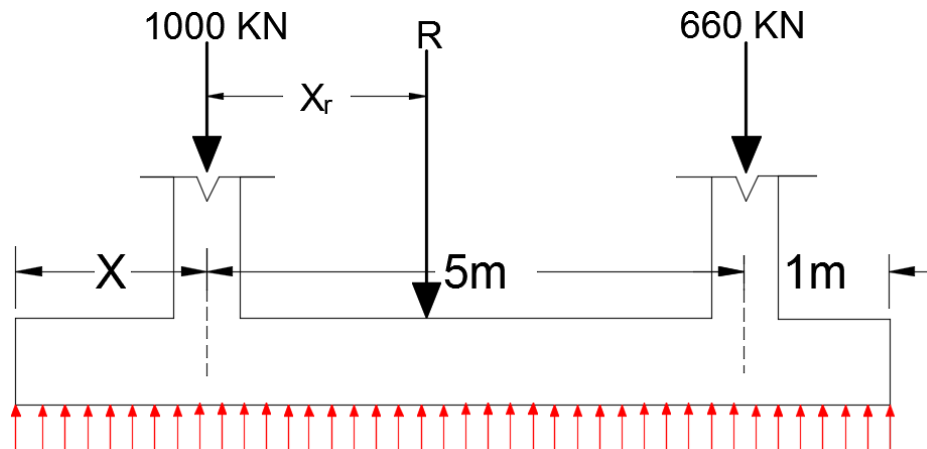
$$Q_1 = 1000 \text{ kN}$$

$$Q_2 = 660 \text{ kN}$$

$$R = Q_1 + Q_2 = 1000 + 660$$

$$R = 1660 \text{ kN}$$

The weight of the foundation and the soil is not given, so we neglect it.





$$\sum M_{c_1} = 0.0 \rightarrow 660 \times 5 = 1660 \times X_r \rightarrow X_r = 1.98 \text{ m}$$

To keep uniform pressure: $X_r + X = \frac{L}{2}$

$$\frac{L}{2} = (5 - 1.98) + 1 = 4.02 \text{ m}$$

$$\frac{L}{2} = 4.02 \text{ m} \rightarrow L = 4.02 \times 2 = 8.04 \text{ m}$$

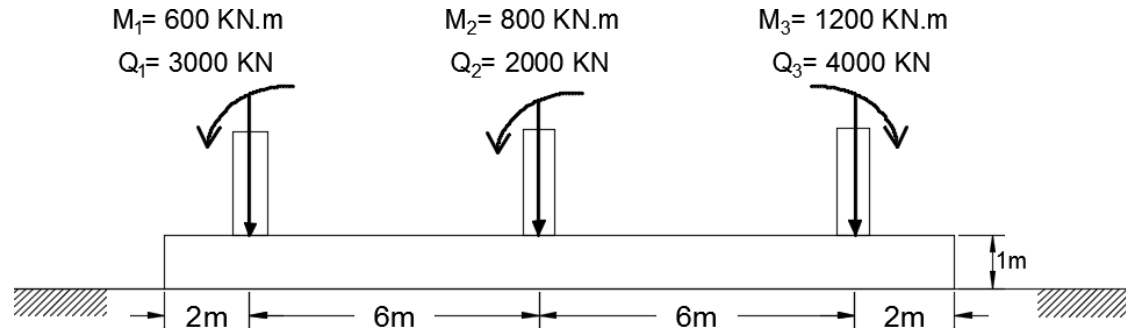
$$X_r + X = \frac{L}{2} \rightarrow 1.98 + X = 4.02 \rightarrow X = 2.04 \text{ m} \checkmark .$$

Calculation of B:

$$A_{\text{req}} = \frac{Q_{\text{service}}}{q_{\text{all,net}}} = B \times L \rightarrow \frac{1660}{140} = B \times 8.04 \rightarrow B = 1.47 \text{ m} \checkmark .$$



3. Determine B_1 and B_2 of a trapezoidal footing for a **uniform** soil pressure of 300 kN/m^2 . (Consider the weight of the footing " $\gamma_{\text{concrete}} = 24 \text{ KN/m}^3$ ").



Solution

$$R = \sum Q = 9000 \text{ kN}$$

$$L = 2 + 6 + 6 + 2 = 16\text{m}$$

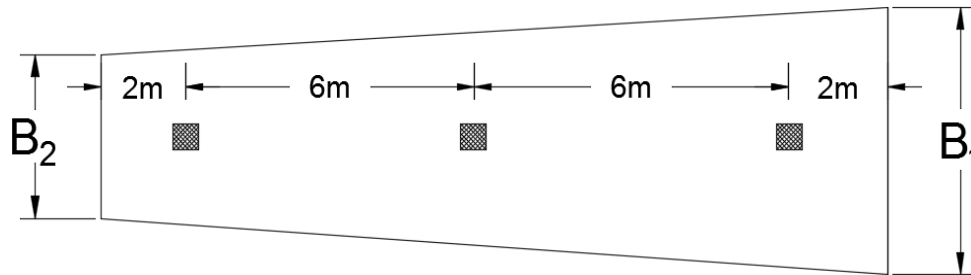
$$\text{Footing Weight} = \text{Footing Volume} \times \gamma_c$$

$$\text{Footing Volume} = \frac{L}{2} (B_1 + B_2) \times 1 = 8(B_1 + B_2)$$

$$\text{Footing Weight} = 8(B_1 + B_2) \times 24 = 192(B_1 + B_2)$$

Note:

$$A_{\text{req}} = \frac{Q_{\text{service,net}}}{q_{\text{all,net}}} = \frac{Q_{\text{service,gross}}}{q_{\text{all,gross}}}$$



$Q_{\text{service,net}}$ = applied column load on the foundation

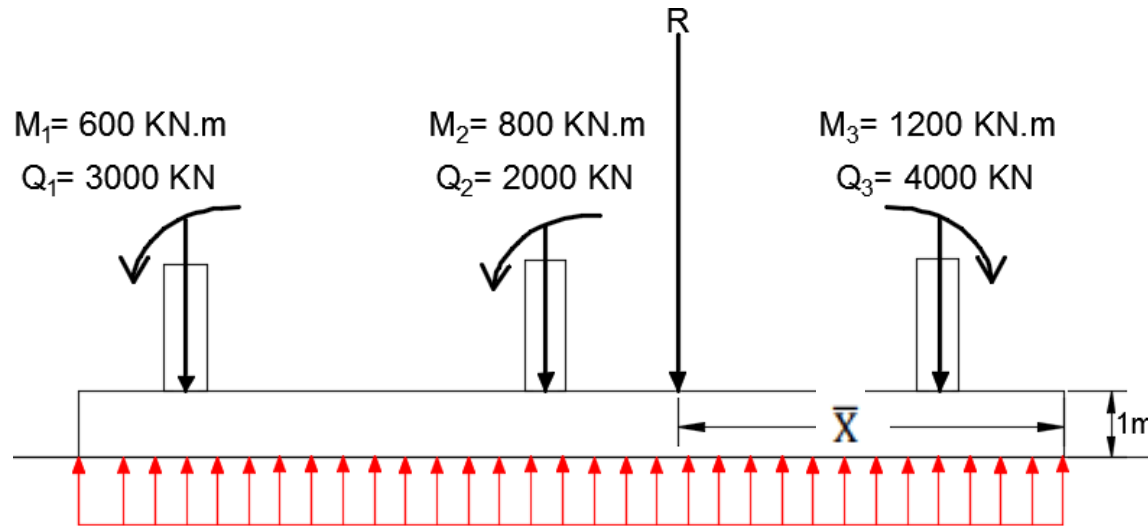
$Q_{\text{service,gross}}$ = Applied column load + footing weight + soil weight

In this problem the footing weight is given and the soil weight is not given, so we neglect it.

The given bearing capacity is allowable gross bearing capacity.

$$A_{\text{req}} = \frac{Q_{\text{service,gross}}}{q_{\text{all,gross}}} = 8(B_1 + B_2) \rightarrow \frac{9000 + 192(B_1 + B_2)}{300}$$

$$B_1 + B_2 = 4.07 \rightarrow B_1 = 4.07 - B_2 \rightarrow \rightarrow \text{Eq. (1)}$$



$$\sum M_{\text{center}} = 0.0 \rightarrow 3000 \times (14 - \bar{X}) + 600 + 2000(8 - \bar{X}) + 800 - 4000 \times (\bar{X} - 2) - 1200 = 0.0 \rightarrow \bar{X} = 7.36 \text{ m}$$

$$\bar{X} = 7.36 = \frac{L}{3} \left(\frac{B_1 + 2B_2}{B_1 + B_2} \right) \rightarrow 7.36 = \frac{16}{3} \left(\frac{B_1 + 2B_2}{B_1 + B_2} \right)$$

$$1.38 = \left(\frac{B_1 + 2B_2}{B_1 + B_2} \right) \rightarrow \text{Eq. (2) substitute from Eq. (1)}$$

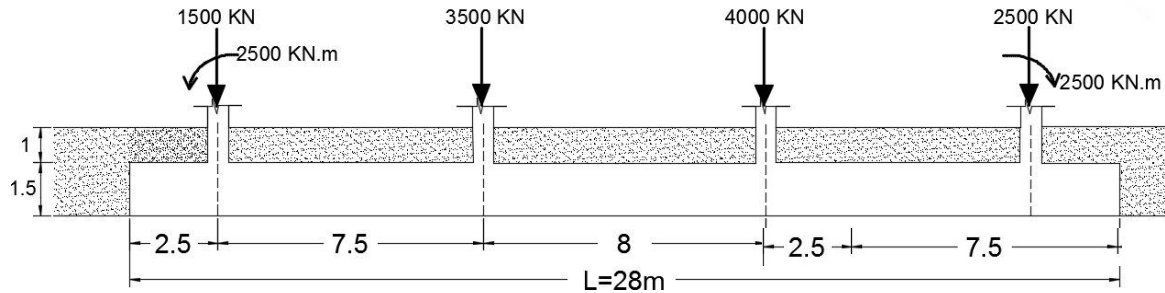
$$1.38 = \left(\frac{4.07 - B_2 + 2B_2}{4.07 - B_2 + B_2} \right) \rightarrow B_2 = 1.55 \text{ m} \checkmark \rightarrow B_1 = 2.52 \text{ m} \checkmark .$$



4. A combined footing consists of four columns as shown in figure, **determine** the width of the rectangular combined footing B.

Allowable gross soil pressure = 255 kN/m^2 .

$\gamma_c = 24 \text{ KN/m}^3$ and $\gamma_{\text{soile}} = 20 \text{ KN/m}^3$



Solution

Note that the foundation is rectangular (as given) but the pressure in this case will not be uniform, so we will design the footing for eccentric loadings.

$$W_f + W_s = 28 \times B \times (1 \times 20 + 1.5 \times 24) = 1568 B$$

$$\sum Q = R_{\text{service,gross}} = 11500 + 1568 B$$

Now calculate the moment at the centroid of the footing:

$$\begin{aligned} \sum M_{\text{@center}} &= 1500 \times (14 - 2.5) + 2500 + 3500 \times 4 - 4000 \times 4 \\ &\quad - 2500(14 - 2.5) - 2500 = -13500 \text{ kN.m} \end{aligned}$$



$$e = \frac{\sum M_{@center}}{\sum Q} = \frac{13500}{11500 + 1568 B}$$

The eccentricity in L direction and since L is so large $e < \frac{L}{6} \rightarrow$

$$q = \frac{Q}{B \times L} \left(1 \pm \frac{6e}{L} \right)$$

$$q_{\max} = \frac{Q}{B \times L} \left(1 + \frac{6e}{L} \right) \quad \text{by equating } q_{\max} \text{ and } q_{\text{all,gross}} \rightarrow$$

$$255 = \frac{11500 + 1568 B}{B \times 28} \left(1 + \frac{6 \left(\frac{13500}{11500 + 1568 B} \right)}{28} \right)$$

$$\rightarrow B = 2.58 \text{ m } \checkmark .$$

Check For B:

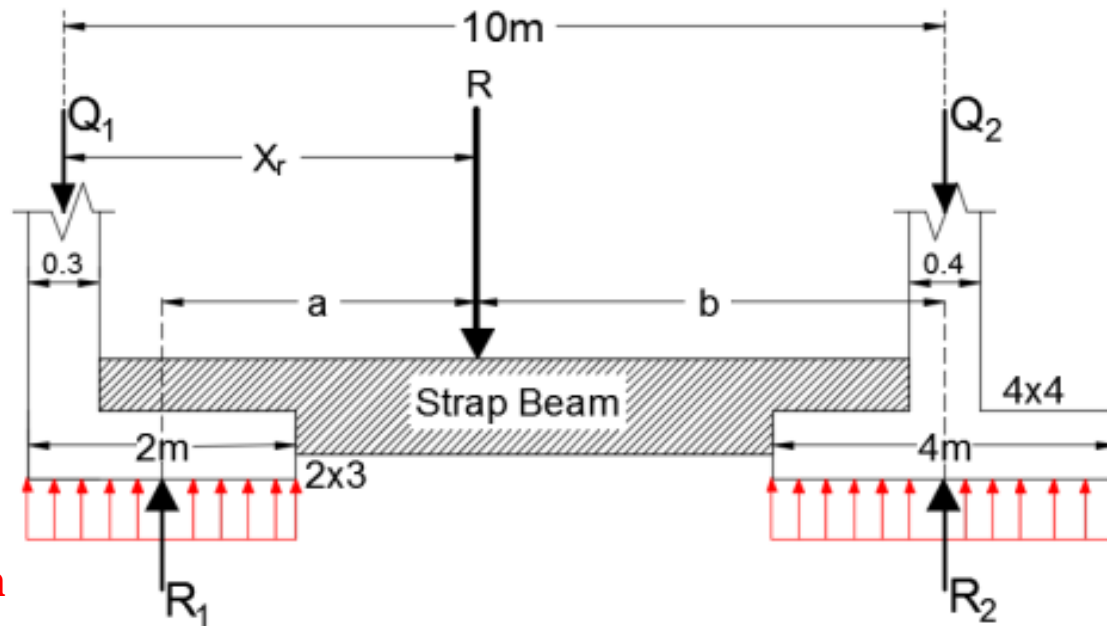
$$e = \frac{13500}{11500 + 1568 \times 2.58} = 0.868 \text{ m}$$

$$\sum Q = 11500 + 1568 \times 2.58 = 15545.4$$

$$q_{\min} = \frac{Q}{B \times L} \left(1 - \frac{6e}{L} \right) = \frac{15545.4}{2.58 \times 28} \left(1 - \frac{6 \times 0.868}{28} \right) = 175.2 \text{ kN/m}^2 > 0 \text{ Ok}$$



5. For the strap footing shown below, if $q_{all,net} = 250 \text{ kN/m}^2$, determine Q_1 and Q_2



Solution

$$R_1 = A_1 \times q_{all,net} = (2 \times 3) \times 250 = 1500 \text{ KN}$$

$$R_2 = A_2 \times q_{all,net} = (4 \times 4) \times 250 = 4000 \text{ KN}$$

$$R = R_1 + R_2 = 1500 + 4000 = 5500 \text{ KN} = Q_1 + Q_2$$

$$a + b = 10 + 0.15 - 1 = 9.15 \text{ m}$$

$$\sum M_{R_2} = 0.0 \text{ (after use of strap)} \rightarrow 1500 \times 9.15 = 5500 \times b \rightarrow b = 2.5 \text{ m}$$

$$\rightarrow a = 9.15 - 2.5 = 6.65 \text{ m} \rightarrow X_r = 6.65 + 1 - 0.15 = 7.5 \text{ m}$$

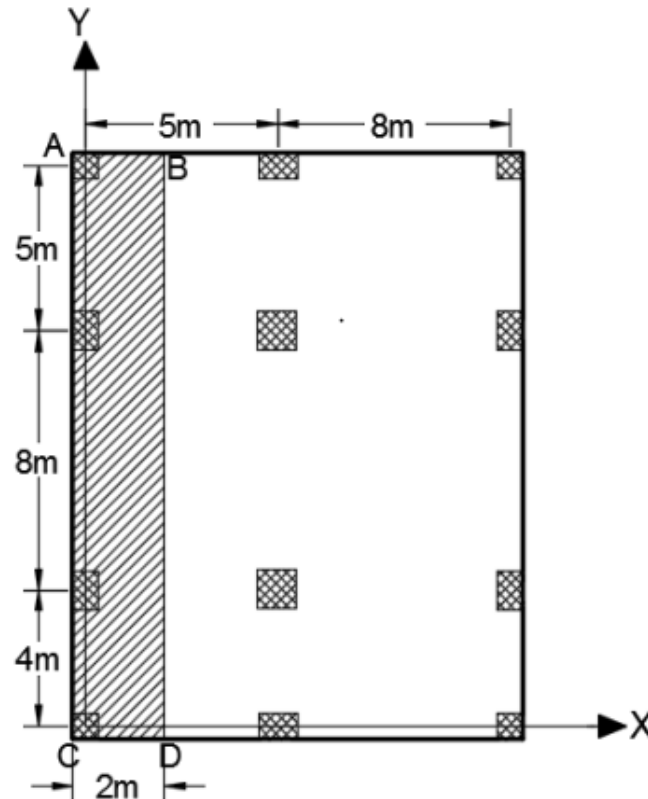
$$\sum M_{@ Q_1} = 0.0 \rightarrow 10Q_2 = 5500 \times 7.5 \rightarrow Q_2 = 4125 \text{ kN} \checkmark.$$

$$\rightarrow Q_1 = R - Q_2 = 5500 - 4125 = 1375 \text{ kN} \checkmark.$$



5. For the shown mat foundation, If $q_{all,net} = 150 \text{ kN/m}^2$.
Check the adequacy of the foundation dimensions.

	Interior Columns	Edge Columns	Corner Columns
Columns Dimensions	60cm x 60cm	60 cm x 40 cm	40 cm x 40 cm
Service Loads	1800 kN	1200 kN	600 kN
Factored Loads	2700 kN	1800 kN	900 kN





Solution

$$B = 5 + 8 + 2 \times 0.2 = 13.4\text{m (horizontal dimension)}$$

$$L = 5 + 8 + 4 + 2 \times 0.2 = 17.4\text{m (vertical dimension)}$$

$$\bar{X} = \frac{13.4}{2} - \frac{0.4}{2} = 6.5\text{m (from y - axis)}$$

$$\bar{Y} = \frac{17.4}{2} - \frac{0.4}{2} = 8.5\text{ m (from x - axis)}$$

$$R = \sum Q_i \text{ (service loads)} = 2 \times 1800 + 6 \times 1200 + 4 \times 600 = 13200\text{ kN}$$

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$
$$= \frac{2 \times 1800 \times 5 + 2 \times 1200 \times 5 + 2 \times 1200 \times 13 + 2 \times 600 \times 13}{13200}$$

$$X_R = 5.82\text{ m (from y - axis)}$$

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$
$$= \frac{2 \times 1200 \times 4 + 1800 \times 4 + 2 \times 1200 \times 12 + 1800 \times 12 + 2 \times 600 \times 17 + 1200 \times 17}{13200}$$

$$Y_R = 8.2\text{ m (from x - axis)}$$



$$e_x = |X_R - \bar{X}| = |5.82 - 6.5| = 0.68$$

$$e_y = |Y_R - \bar{Y}| = |8.2 - 8.5| = 0.3\text{m}$$

$$M_x = e_y \times \sum Q_i = 0.3 \times 13200 = 3960 \text{ kN.m}$$

$$M_y = e_x \times \sum Q_i = 0.68 \times 13200 = 8976 \text{ kN.m}$$

$$I_x = \frac{B L^3}{12} = \frac{13.4 \times 17.4^3}{12} = 5882.6 \text{ m}^4$$

$$I_y = \frac{L B^3}{12} = \frac{17.4 \times 13.4^3}{12} = 3488.85 \text{ m}^4$$

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y \rightarrow q = \frac{13200}{13.4 \times 17.4} \pm \frac{8976}{3488.85} X \pm \frac{3960}{5882.6} Y$$

$$q = 56.61 \pm 2.57 X \pm 0.67 Y$$

$$q_{\max} = 56.61 + 2.57 X + 0.67 Y$$

$$q_{\min} = 56.61 - 2.57 X - 0.67 Y$$

$$\rightarrow q_{\max} = 56.61 + 2.57 \times 6.7 + 0.67 \times 8.7 = 79.66 \text{ kN/m}^2$$

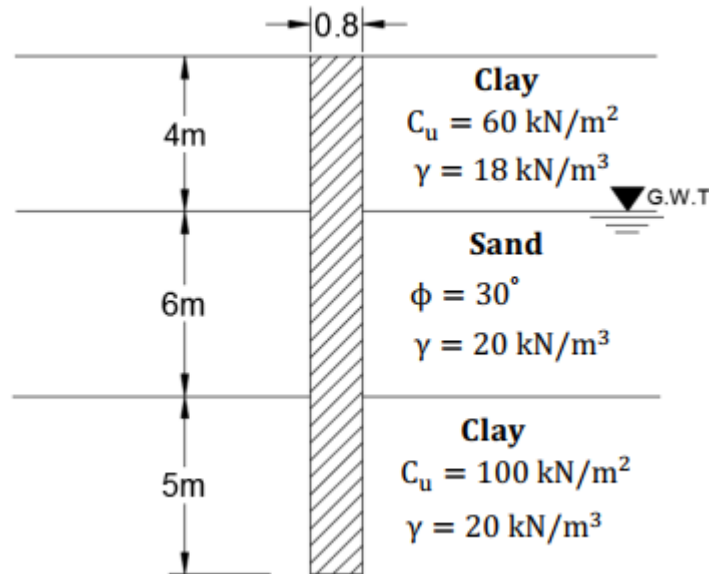
$$\rightarrow q_{\min} = 56.61 - 2.57 \times 6.7 - 0.67 \times 8.7 = 33.56 \text{ kN/m}^2$$

$$q_{\max} = 79.66 < q_{\text{all,net}} = 150 \rightarrow \text{Ok}$$

$$q_{\min} = 33.56 > 0.0 \rightarrow \text{Ok} \quad \text{The foundation Dimensions are adequate} \checkmark .$$



6. Determine the ultimate load capacity of the 800 mm diameter concrete bored pile given in the figure below.



Solution

Calculation of Q_P :

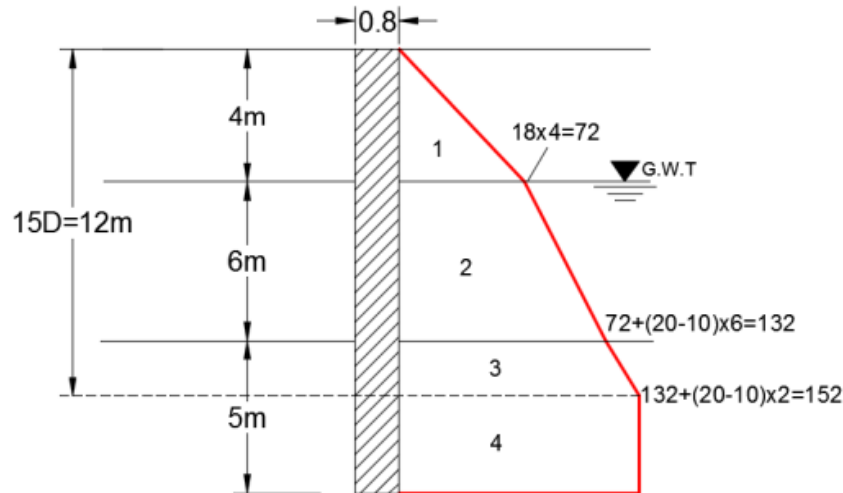
$$Q_P = A_P \times c_u \times N_c^* \quad N_c^* = 9 \text{ (pure clay } \phi = 0.0)$$



Calculation of Q_s :

For sand:

$$15D = 15 \times 0.8 = 12\text{m}$$



$$Q_s = P \times \sum (\tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i) \times L_i$$

$$\sigma'_{v,av,2} = \frac{72 + 132}{2} = 102 \text{ kN/m}^2$$

$$L_2 = 6\text{m}$$

$$\phi_2 = 30^\circ$$

$$K_2 = 1 - \sin\phi_2 = 1 - \sin 30 = 0.5$$

$$\rightarrow Q_{s,sand} = 2.51 \times (\tan(0.8 \times 30) \times 102 \times 0.5) \times 6 = 342 \text{ kN}$$



For Clay:

$$Q_s = P \times L \times f_{av}$$

$$f_{av} = \lambda \times (\sigma'_{v,av} + 2 c_{u,av})$$

$$P = 2.51 \text{m} \quad L = 4 + 6 + 5 = 15 \text{m}$$

$$\lambda = 0.2$$

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

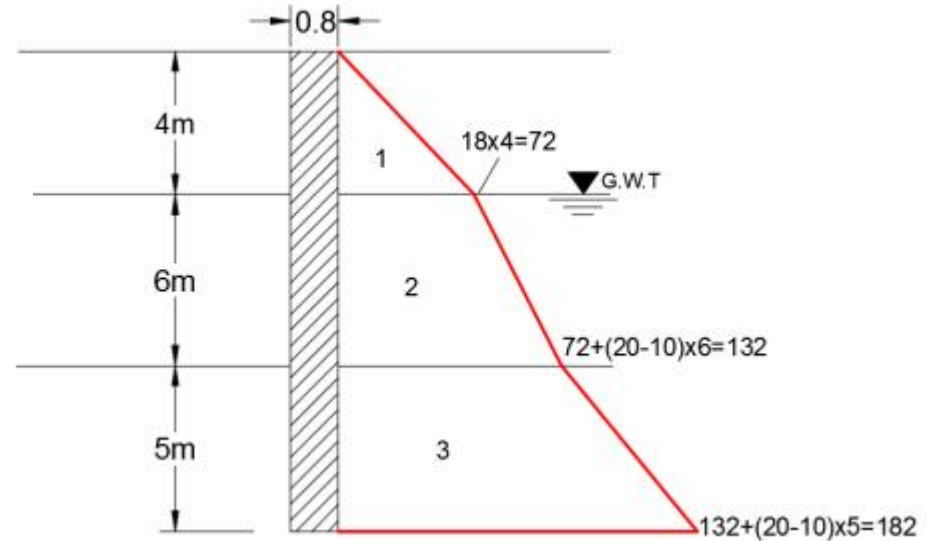
$$c_{u,av} = \frac{4 \times 60 + 6 \times 0 + 5 \times 100}{15} = 49.33$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \dots + A_n}{L}$$

$$A_1 = \frac{1}{2} \times 72 \times 4 = 144$$

$$A_2 = \frac{1}{2} \times (72 + 132) \times 6 = 612$$

$$A_3 = \frac{1}{2} \times (132 + 182) \times 5 = 785$$





$$\sigma'_{v,av} = \frac{144 + 612 + 785}{15} = 102.73 \text{ kN/m}^2$$

$$f_{av} = 0.2 \times (102.73 + 2 \times 49.33) = 40.28$$

$$\rightarrow Q_{s,clay} = 2.51 \times 15 \times 40.28 = 1516.54 \text{ kN}$$

If we want to use α Method:

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

For layer (1)

$$c_{u,1} = 60 \rightarrow \frac{c_{u,1}}{p_{atm}} = \frac{60}{100} = 0.6 \rightarrow \alpha_1 = 0.62$$

For layer (2)

$$c_{u,2} = 0.0 \rightarrow \frac{c_{u,2}}{p_{atm}} = \frac{0}{100} = 0 \rightarrow \alpha_2 = 0$$

For layer (3)

$$c_{u,3} = 100 \rightarrow \frac{c_{u,3}}{p_{atm}} = \frac{100}{100} = 1 \rightarrow \alpha_3 = 0.48$$

$$Q_{s,clay} = 2.51 \times [(0.62 \times 60 \times 4) + 0 + (0.48 \times 100 \times 5)] = 975.88 \text{ kN}$$

$$Q_{s,total} = Q_{s,sand} + Q_{s,clay}$$



$$Q_{s,\text{total}} = 342 + 1516.54 = 1858.54 \text{ kN (when using } \lambda \text{ - method)}$$

$$Q_{s,\text{total}} = 342 + 975.88 = 1317.88 \text{ kN (when using } \alpha \text{ - method)}$$

$$Q_u = Q_p + Q_{s,\text{total}}$$

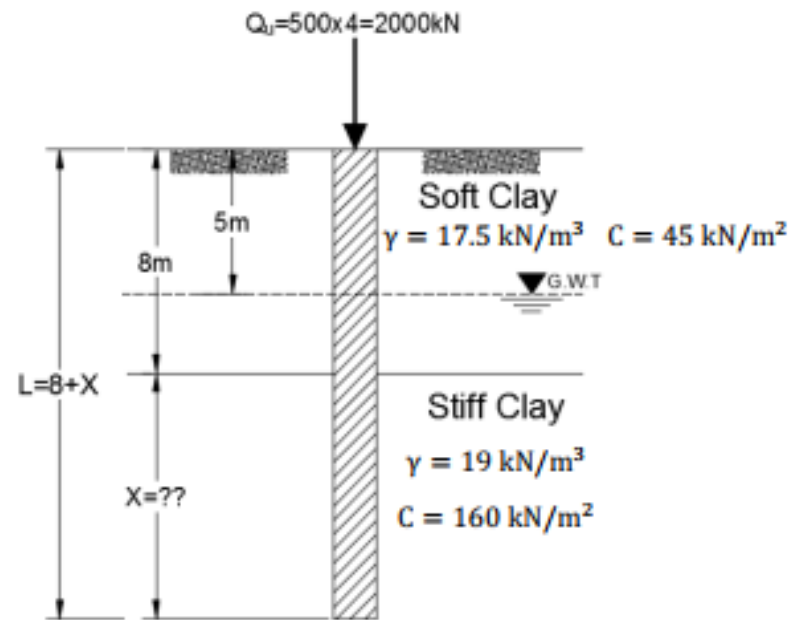
$$Q_u = 452.4 + 1858.54 = 2310.94 \text{ kN (when using } \lambda \text{ - method) } \checkmark .$$

$$Q_u = 452.4 + 1317.88 = 1770.28 \text{ kN (when using } \alpha \text{ - method) } \checkmark .$$



7. A pile is driven through a soft cohesive deposit overlying a stiff clay, the average un-drained shear strength in the soft clay is 45 kPa. and in the lower deposit the average un-drained shear strength is 160 kPa. The water table is 5 m below the ground and the stiff clay is at 8 m depth. The unit weights are 17.5 kN/m^3 and 19 kN/m^3 for the soft and the stiff clay respectively. **Estimate the length** of 500 mm diameter pile to carry a load of 500 kN with a safety factor of 4. Using (a). α – method (b). λ – method

Solution





$$Q_{all} = 500 \text{ kN} , \text{ FS} = 4 \rightarrow Q_u = 500 \times 4 = 2000 \text{ KN}$$

$$Q_u = Q_P + Q_s$$

Calculation of Q_P :

$$Q_P = A_P \times c_u \times N_c^* \quad N_c^* = 9 \text{ (pure clay } \phi = 0.0)$$

$$Q_P = 0.196 \times 160 \times 9 = 282.24 \text{ KN}$$

Calculation of Q_s :

(a). α - method

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

$$P = \pi \times D = \pi \times 0.5 = 1.57 \text{ m}$$

For layer (1)

$$c_{u,1} = 45 \rightarrow \frac{c_{u,1}}{p_{atm}} = \frac{45}{100} = 0.45 \rightarrow \alpha_1 = 0.71 \text{ (by interpolation from table)}$$

For layer (2)

$$c_{u,2} = 160 \rightarrow \frac{c_{u,2}}{p_{atm}} = \frac{160}{100} = 1.6 \rightarrow \alpha_2 = 0.38$$

$$Q_s = 1.57 \times [(0.71 \times 45 \times 8) + (0.38 \times 160 \times X)] = 401.3 + 95.45X \text{ Kn}$$



But $Q_u = 2000 \rightarrow 2000 = 683.54 + 95.45X \rightarrow X = 13.8\text{m}$
 $\rightarrow L = 8 + 13.8 = 21.8 \cong 22\text{m} \checkmark$.

(b). λ - method

$$Q_s = P \times L \times f_{av}$$

$$f_{av} = \lambda \times (\sigma'_{v,av} + 2 c_{u,av})$$

$$P = 1.57\text{m} \quad L = 8 + X$$

Assume $X = 7\text{m} \rightarrow L = 8 + 7 = 15$

$$\lambda = 0.2$$

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

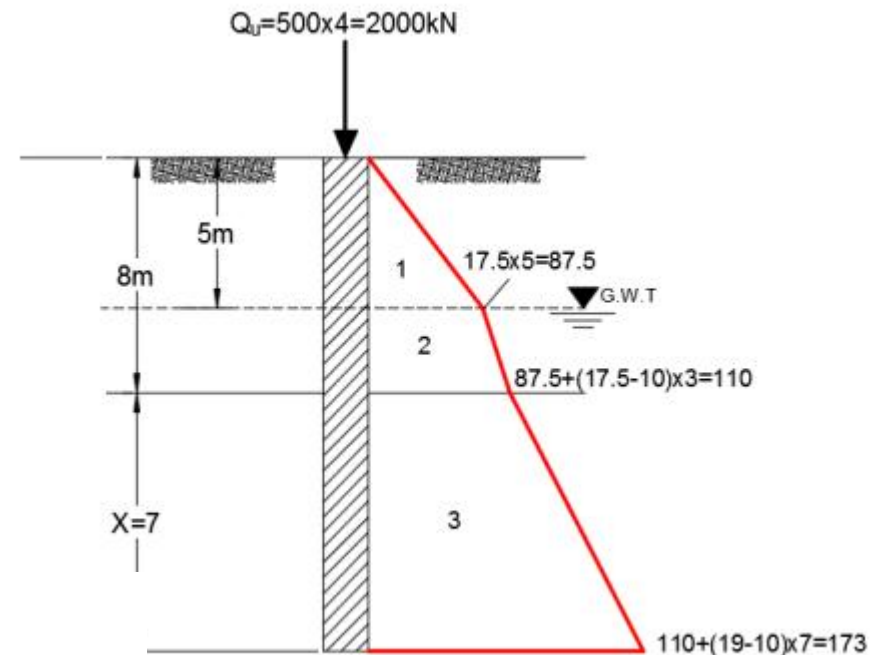
$$c_{u,av} = \frac{8 \times 45 + 7 \times 160}{15} = 98.67 \text{ kN/m}^2$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \dots + A_n}{L}$$

$$\sigma'_{v,av} = \frac{218.75 + 296.25 + 990.5}{15} = 100.36 \text{ kN/m}^2$$

$$f_{av} = 0.2 \times (100.36 + 2 \times 98.67) = 59.54$$

$$\rightarrow Q_s = 1.57 \times 15 \times 59.54 = 1402.167 \text{ kN}$$





$Q_u = Q_p + Q_s = 282.24 + 1402.167 = 1684.4 < 2000 \rightarrow \rightarrow \rightarrow$ We need to increase L to be closed from 2000

Try $X = 12 \text{ m} \rightarrow L = 8 + 12 = 20$

$$\lambda = 0.173$$

$$c_{u,av} = \frac{8 \times 45 + 12 \times 160}{20} = 114 \text{ kN/m}^2$$

$$\sigma'_{v,av} = \frac{218.75 + 296.25 + 1968}{20} = 124.15 \text{ kN/m}^2$$

$$f_{av} = 0.173 \times (124.15 + 2 \times 114) = 60.92$$

$$\rightarrow Q_s = 1.57 \times 20 \times 60.92 = 1912.94 \text{ kN}$$

$$\begin{aligned} Q_u &= Q_p + Q_s \\ &= 282.24 + 1912.94 = 2195.12 > 2000 \end{aligned}$$

