

## Ex.1: Slender Concrete Column Design in Sway Frame Buildings

Evaluate slenderness effect for the first story exterior column in a sway frame multistory reinforced concrete building.


## Ex.1: Slender Concrete Column Design in Sway Frame Buildings

The clear height of the first story columns is 4.7 m , and is 2.7 m for all of the other stories. Lateral load effects on the building are governed by wind forces.


Direction of analysis

## Ex.1: Slender Concrete Column Design in Sway Frame Buildings



## Ex.1: 1. Design Data

Design Code: Building Code Requirements for Structural Concrete (ACI 318M-14)
$f_{c}^{\prime}=40 \mathrm{MPa}$ for columns in the bottom two stories
$f^{\prime}{ }_{c}=27 \mathrm{MPa}$ elsewhere
$f_{y}=415 \mathrm{MPa}$
$E_{s}=200000 \mathrm{MPa}$
$w_{c}=24 \mathrm{kN} / \mathrm{m}^{3}$
Slab thickness = 15 cm
Exterior Columns $=45 \mathrm{~cm} \times 45 \mathrm{~cm}$
Interior Columns $=50 \mathrm{~cm} \times 50 \mathrm{~cm}$
Beams $=45 \mathrm{~cm} \times 75 \mathrm{~cm} \times 9.75 \mathrm{~m}$ (center to center)
Floor superimposed dead load $=0.96 \mathrm{kN} / \mathrm{m}^{2}$
Floor live load $=3.83 \mathrm{kN} / \mathrm{m}^{2}$
Roof superimposed dead load $=1.20 \mathrm{kN} / \mathrm{m}^{2}$
Roof live load $=1.44 \mathrm{kN} / \mathrm{m}^{2}$
Wind loads computed according to ASCE 7-10

## Ex.1: 1. Design Data

Total building loads in the first story:

| Table 1-Total building factored loads |  |  |  |
| :--- | :---: | :--- | :---: |
| ASCE 7-10 <br> Reference | No. | Load combination | $\boldsymbol{\Sigma} \boldsymbol{P}_{\iota^{\prime}} \mathrm{kN}$ |
| $2.3 .2-1$ | 1 | 1.4 D | 48886 |
| $2.3 .2-2$ | 2 | $1.2 \mathrm{D}+1.6 \mathrm{~L}+0.5 \mathrm{~L}_{\mathrm{r}}$ | 50710 |
| $2.3 .2-3$ | 3 | $1.2 \mathrm{D}+0.5 \mathrm{~L}+1.6 \mathrm{~L}_{\mathrm{r}}$ | 46524 |
|  | 4 | $1.2 \mathrm{D}+1.6 \mathrm{Lr}+0.8 \mathrm{~W}$ | 43957 |
|  | 5 | $1.2 \mathrm{D}+1.6 \mathrm{Lr}-0.8 \mathrm{~W}$ | 43957 |
| $2.3 .2-4$ | 6 | $1.2 \mathrm{D}+0.5 \mathrm{~L}+0.5 \mathrm{~L}_{\mathrm{r}}+1.6 \mathrm{~W}$ | 44927 |
|  | 7 | $1.2 \mathrm{D}+0.5 \mathrm{~L}+0.5 \mathrm{~L}_{\mathrm{r}}-1.6 \mathrm{~W}$ | 44927 |
| $2.3 .2-6$ | 8 | $0.9 \mathrm{D}+1.6 \mathrm{~W}$ | 31427 |
|  | 9 | $0.9 \mathrm{D}-1.6 \mathrm{~W}$ | 31427 |

## Ex.1: 1. Design Data

Total building loads in the first story:
$V_{s}$ is the horizontal story shear in the first story corresponding to the wind loads.
Shear due to wind loads in the first story $V_{s}=1346 \mathrm{kN}$
$\Delta$ is the first-order relative drift between the top and bottom of the first story due to $V_{s}$.
Story drift due to $V_{s}$ in the first story $\quad \Delta=20.25 \mathrm{~mm}$

## Ex.1: 1. Design Data

1. Factored Axial Loads and Bending Moments for the Design Column
1.1. Service loads

| Table 2 - Exterior column service loads |  |  |  |
| :---: | :---: | :---: | :---: |
| Load Case | Axial Load <br> kN | Bending Moment, kN.m |  |
|  | Top | Bottom |  |
| Dead, D | 1258.85 | -47.32 | 49.89 |
| Live, L | 190.83 | -15.19 | 16 |
| Roof Live, $\mathrm{L}_{\mathrm{r}}$ | 44.93 | 0 | 0 |
| Wind (+Dir), W (S-N) | -40.03 | 64.81 | -62.5 |
| Wind (-Dir), W (N-S) | 40.03 | -64.81 | 62.5 |

ملاحظة 1: إشارة العزم تدل على جهة شد الألياف و ليس على جهة الدوران.
ملاحظة 2: الإشارة السالبة للقوة المحورية تعني الشد، بينما الموجبة تعني الضغط.

## Ex.1: 2. Sway or Nonsway Frame Designation

ACl 318M-14 (6.6.4.3): Columns and stories in structures are considered as nonsway frames if the increase in column end moments due to second-order effects does not exceed 5\% of the first-order end moments, or the stability index for the story $(Q)$ does not exceed 0.05 .

$$
Q=\frac{\left(\sum P_{u}\right) \Delta_{0}}{V_{u s} l_{c}} \leq 0.05 \quad \text { ACI 318M-14 (Eq. 6.6.4.4.1) }
$$

ACI 318-M14 (6.6.4.4.1 and R6.6.4.3): $\sum P_{u}$ is the total vertical load in the first story corresponding to the lateral loading case for which $\sum P_{u}$ is greatest (without the wind loads, which would cause compression in some columns and tension in others and thus would cancel out).

ACI 318-14 (6.6.4.4.1 and R6.6.4.3): $V_{u s}$ is the factored horizontal story shear in the first story corresponding to the wind loads, and $\Delta_{0}$ is the first-order relative deflection between the top and bottom of the first story due to $V_{u s}$.

## Ex.1: 2. Sway or Nonsway Frame Designation

Thus, the frame at the first story level.
From Table 1, load combinations (2.3.2-4 No. 6 and 7 ) provide the greatest value of $\sum P_{u}$

$$
\begin{aligned}
& \sum P_{u}=44927 \mathrm{kN} \\
& V_{u s}=1.6 V_{s}=1.6 \times 1346=2153.6 \mathrm{kN} \\
& \Delta_{0}=1.6 \Delta=1.6 \times 20.25=32.4 \mathrm{~mm} \\
& l_{c}=4.7+\frac{0.75}{2}=5.075 \mathrm{~m} \\
& Q=\frac{\left(\sum P_{u}\right) \Delta_{0}}{V_{u s} l_{c}}=\frac{44927 \times 32.4}{2153.6 \times 5075}=0.133>0.05
\end{aligned}
$$

Thus, the frame at the first story level is considered sway.


## Ex.1: 3. Determine Slenderness Effects

$$
\begin{aligned}
I_{c} & =0.7 \frac{c^{4}}{12}=0.7 \times \frac{45^{4}}{12}=239203.13 \mathrm{~cm}^{4} \\
E_{c} & =w_{c}^{1.5} 0.043 \sqrt{f_{c}^{\prime}}=2400^{1.5} \times 0.043 \times \sqrt{40} \\
& =31975.35 \mathrm{MPa}
\end{aligned}
$$

For the column below level 2

$$
\begin{aligned}
\frac{E_{c} I_{c}}{l_{c}} & =\frac{31975.35 \times 239203.13 \times 10^{4}}{5075} \times 10^{-6} \\
& =15071.14 \mathrm{kN.m}
\end{aligned}
$$

For the column above level 2

$$
\frac{E_{c} I_{c}}{l_{c}}=\frac{31975.35 \times 239203.13 \times 10^{4}}{2700+750} \times 10^{-6}
$$



$$
=22169.87 \mathrm{kN} . \mathrm{m}
$$

## Ex.1: 3. Determine Slenderness Effects

$I_{b}=0.35 \frac{b_{w} h_{b}^{3}}{12}=0.35 \times \frac{45 \times 75^{3}}{12}=553710.94 \mathrm{~cm}^{4}$
$I_{b}$ for $T$ beams can be closely approximated as 2 times $I_{g}$ for the web.
$E_{b}=w_{c}^{1.5} 0.043 \sqrt{f_{c}^{\prime}}=2400^{1.5} \times 0.043 \times \sqrt{27}=26270.43 \mathrm{MPa}$

For beams framing into the columns:
Only one beam framing to this column in N-S direction

$$
\frac{E_{b} I_{b}}{l_{b}}=\frac{26270.43 \times 553710.94 \times 10^{4}}{9750} \times 10^{-6}=14919.20 \mathrm{kN.m}
$$

$\Psi_{A}=\frac{\sum E_{c} I_{c} / L_{c}}{\sum E_{b} I_{b} / L_{b}}=\frac{15071.14+22169.87}{14919.20}=2.5$
$\Psi_{B}=1 \quad$ (Column essentially fixed at base)

## Ex.1: 3. Determine Slenderness Effects

Effective Length Factor (k) Calculations for Exterior Columns with One Beam Framing into them in the Direction of Analysis (Sway Frame)
$\Psi_{A}=2.5$
$\Psi_{B}=1$

Using Figure R6.2.5 from ACI 318M-14: $k=1.5$ as shown in the figure for the exterior columns with one beam framing into them in the directions of analysis.


عوامل الطول الفعال للعناصر المضغوطة في الإطارات المنزاحة جانبياً

## Ex.1: 3. Determine Slenderness Effects

$r=$ radius of gyration $=\left\{\begin{array}{l}\text { (a) } \sqrt{\frac{I_{g}}{A_{g}}} \\ \text { or } \\ \text { (b) } 0.3 \times c_{1}\end{array}\right.$
$r=\sqrt{\frac{I_{g}}{A_{g}}}=\sqrt{\frac{45^{4} / 12}{45^{2}}}=12.99 \mathrm{~cm}$

$$
\frac{k l_{u}}{r}=\frac{1.5 \times 470}{12.99}=54.27>22 \quad \rightarrow \text { Consider Slenderness }
$$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

A detailed calculation for load combination 5 (gravity minus wind) is shown below to illustrate the procedure.

$$
\begin{gathered}
M_{1}=M_{1 n s}+\delta_{s} M_{1 s} \\
M_{2}=M_{2 n s}+\delta_{s} M_{2 s} \\
\delta_{s} M_{s}=\left\{\begin{array}{c}
\text { (a) } \frac{M_{s}}{1-Q} \\
\text { (b) } \frac{M_{S}}{1-\frac{\sum P_{u}}{0.75 \sum P_{c}}} \\
\text { (c) Second-order elastic analysis }
\end{array}\right.
\end{gathered}
$$

ACI 318-14 (6.6.4.6.2(b)) will be used. However, (a) and (c) can also be used to calculate the moment magnifier.
$\sum P_{u}$ is the summation of all the factored vertical loads in the first story, and $\sum P_{c}$ is the summation of the critical buckling load for all sway-resisting columns in the first story.

## Ex.1: 4. Moment Magnification at Ends of Compression Member

$$
\begin{aligned}
& P_{c}=\frac{\pi^{2}(E I)_{e f f}}{\left(k l_{u}\right)^{2}} \\
& \left(\text { (a) } \frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d s}}\right. \\
& (E I)_{e f f}=\left\{(b) \frac{0.4 E_{c} I_{g}}{1+\beta_{d s}}\right. \\
& \text { (c) } \frac{E_{c} I}{1+\beta_{d s}} \\
& I \text { from ACI 318M-14 (Table 6.6.3.1.1(b)) }
\end{aligned}
$$

There are three options for calculating the effective flexural stiffness of slender concrete columns ( $E \Lambda_{\text {eff }}$ The first equation provides accurate representation of the reinforcement in the section and will be used in this example.

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(1) For exterior columns with one beam framing into them in the direction of analysis
( 12 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )


## Ex.1: 4. Moment Magnification at Ends of Compression Member

(1) For exterior columns with one beam framing into them in the direction of analysis
( 12 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )
$E I=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d s}}$
$I_{g}=\frac{c^{4}}{12}=\frac{45^{4}}{12}=341718.75 \mathrm{~cm}^{4}$
$E_{c}=w_{c}^{1.5} 0.043 \sqrt{f_{c}^{\prime}}=2400^{1.5} \times 0.043 \times \sqrt{40}=31975.35 \mathrm{MPa}$

ACl 318-14 (6.6.3.1.1) : $\beta_{d s}$ is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination.
The maximum factored sustained shear in this example is equal to zero leading to $\beta_{d s}=0$.

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(1) For exterior columns with one beam framing into them in the direction of analysis ( 12 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )

With 8-\#20 reinforcement equally distributed on all sides and $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ column section:
$I_{s e}=6 \times \frac{\pi \times 2^{2}}{4} \times(22.5-5)^{2}$
$I_{s e}=5772.68 \mathrm{~cm}^{4}$


## Ex.1: 4. Moment Magnification at Ends of Compression Member

(1) For exterior columns with one beam framing into them in the direction of analysis
( 12 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )

$$
E I=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d s}}
$$

$E I=\frac{0.2 \times 31975.35 \times 341718.75 \times 10^{4}+200000 \times 5772.68 \times 10^{4}}{1+0}$
$E I=13765.64 \times 10^{9} \mathrm{~N} . \mathrm{mm}^{2}$
$k=1.5$ (calculated previously)
$P_{c 1}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 13765.64 \times 10^{9}}{(1.5 \times 4700)^{2}}=2733.49 \times 10^{3} \mathrm{~N}=2733.49 \mathrm{kN}$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(2) For exterior columns with two beams framing into them in the direction of analysis ( 4 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )


## Ex.1: 4. Moment Magnification at Ends of Compression Member

(2) For exterior columns with two beams framing into them in the direction of analysis ( 4 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )
$\Psi_{A}=\frac{\sum E_{c} I_{c} / L_{c}}{\sum E_{b} I_{b} / L_{b}}=\frac{15071.14+22169.87}{2 \times 14919.20}=1.25$
$\Psi_{B}=1 \quad$ (Column essentially fixed at base)

Effective Length Factor (k) Calculations for Exterior Columns with two Beams Framing into them in the Direction of Analysis (Sway Frame)

Using Figure R6.2.5 from $\mathrm{ACl} 318 \mathrm{M}-14$ : $k=1.35$ as shown in the figure for the exterior columns with two beams framing into them in the directions of analysis.

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(2) For exterior columns with two beams framing into them in the direction of analysis ( 4 columns $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ )
$k=1.35$
$P_{c 2}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 13765.64 \times 10^{9}}{(1.35 \times 4700)^{2}}=3374.68 \times 10^{3} \mathrm{~N}=3374.68 \mathrm{kN}$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(3) For interior columns with two beams framing into them in the direction of analysis ( 8 columns $50 \mathrm{~cm} \times 50 \mathrm{~cm}$ )


## Ex.1: 4. Moment Magnification at Ends of Compression Member

(3) For interior columns with two beams framing into them in the direction of analysis ( 8 columns $50 \mathrm{~cm} \times 50 \mathrm{~cm}$ )

$$
\begin{aligned}
& I_{c}=0.7 \frac{c^{4}}{12}=0.7 \times \frac{50^{4}}{12}=364583.33 \mathrm{~cm}^{4} \\
& E_{c}=w_{c}^{1.5} 0.043 \sqrt{f_{c}^{\prime}}=2400^{1.5} \times 0.043 \times \sqrt{40}=31975.35 \mathrm{MPa}
\end{aligned}
$$

For the column below level 2

$$
\frac{E_{c} I_{c}}{l_{c}}=\frac{31975.35 \times 364583.33 \times 10^{4}}{5075} \times 10^{-6}=22970.80 \mathrm{kN.m}
$$

For the column above level 2

$$
\frac{E_{c} I_{c}}{l_{c}}=\frac{31975.35 \times 364583.33 \times 10^{4}}{2700+750} \times 10^{-6}=33790.38 \mathrm{kN} . \mathrm{m}
$$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(3) For interior columns with two beams framing into them in the direction of analysis ( 8 columns $50 \mathrm{~cm} \times 50 \mathrm{~cm}$ )
$I_{b}=0.35 \frac{b_{w} h_{b}^{3}}{12}=0.35 \times \frac{45 \times 75^{3}}{12}=553710.94 \mathrm{~cm}^{4}$
$E_{b}=w_{c}^{1.5} 0.043 \sqrt{f_{c}^{\prime}}=2400^{1.5} \times 0.043 \times \sqrt{27}=26270.43 \mathrm{MPa}$

For beams framing into the columns:
Two beams framing to this column in $\mathrm{N}-\mathrm{S}$ direction

$$
\begin{aligned}
& \frac{E_{b} I_{b}}{l_{b}}=\frac{26270.43 \times 553710.94 \times 10^{4}}{9750} \times 10^{-6}=14919.20 \mathrm{kN} . \mathrm{m} \\
& \Psi_{A}=\frac{\sum E_{c} I_{c} / L_{c}}{\sum E_{b} I_{b} / L_{b}}=\frac{22970.80+33790.38}{2 \times 14919.20}=1.90 \\
& \Psi_{B}=1 \quad \text { (Column essentially fixed at base) }
\end{aligned}
$$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(3) For interior columns with two beams framing into them in the direction of analysis ( 8 columns $50 \mathrm{~cm} \times 50 \mathrm{~cm}$ )

Effective Length Factor ( $k$ ) Calculations for Exterior Columns with One Beam Framing into them in the Direction of Analysis (Sway Frame)
$\Psi_{A}=1.9$
$\Psi_{B}=1$
Using Figure R6.2.5 from ACI 318M-14: $k=1.44$ as shown in the figure for the interior columns with two beams framing into them in the directions of analysis.


عوامل الطول الفعال للعناصر المضِغوطة في
الإطارات المنزاحة جانبياً

## Ex.1: 4. Moment Magnification at Ends of Compression Member

(3) For interior columns with two beams framing into them in the direction of analysis ( 8 columns $50 \mathrm{~cm} \times 50 \mathrm{~cm}$ )

With 8-\#20 reinforcement equally distributed on all sides and $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ column section:
$I_{s e}=6 \times \frac{\pi \times 2^{2}}{4} \times(25-5)^{2}$
$I_{s e}=7539.82 \mathrm{~cm}^{4}$
$I_{g}=\frac{c^{4}}{12}=\frac{50^{4}}{12}=520833.33 \mathrm{~cm}^{4}$
$E I=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d s}}$


## Ex.1: 4. Moment Magnification at Ends of Compression Member

(3) For interior columns with two beams framing into them in the direction of analysis ( 8 columns $50 \mathrm{~cm} \times 50 \mathrm{~cm}$ )
$E I=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d s}}$
$E I=\frac{0.2 \times 31975.35 \times 520833.33 \times 10^{4}+200000 \times 7539.82 \times 10^{4}}{1+0}$
$E I=48387.30 \times 10^{9} \mathrm{~N} . \mathrm{mm}^{2}$
$k=1.44 \quad$ (calculated previously)
$P_{c 3}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 48387.30 \times 10^{9}}{(1.44 \times 4700)^{2}}=10425.82 \times 10^{3} \mathrm{~N}=10425.82 \mathrm{kN}$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

$$
\begin{aligned}
& \sum P_{c}=n_{1} \times P_{c 1}+n_{2} \times P_{c 2}+n_{3} \times P_{c 3} \\
& \sum P_{c}=12 \times 2733.49+4 \times 3374.68+8 \times 10425.82 \\
& \sum P_{c}=129707.16 \mathrm{kN}
\end{aligned}
$$

$$
\sum P_{u}=43957 \mathrm{kN} \quad \text { (Table 1) load combination } 5 \text { (gravity plus wind) }
$$

$$
\delta_{s}=\frac{1}{1-\frac{\sum P_{u}}{0.75 \sum P_{c}}} \geq 1
$$

$$
\delta_{s}=\frac{1}{1-\frac{43957}{0.75 \times 129707.16}}=1.82
$$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

Table 3 - Total building factored loads

| ASCE 7-10 <br> Reference | Load combination |  | $\mathrm{P}_{\mathrm{u}}$ | $\mathrm{M}_{\mathrm{u} \text {, top }}$ | $\mathrm{M}_{\mathrm{u}, \mathrm{bot}}$ | $\left(\mathrm{M}_{\mathrm{u}, \text { top }}\right)_{\mathrm{ns}}$ | $\left(\mathrm{M}_{\mathrm{u}, \mathrm{bot}}\right)_{\mathrm{ns}}$ | $\left(\mathrm{M}_{\mathrm{u}, \text { top }}\right)_{\mathrm{s}}$ | $\left(\mathrm{M}_{\mathrm{u}, \text { bot }}\right)_{\mathrm{s}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | kN | kN.m | kN.m | kN.m | kN.m | kN.m | kN.m |
| 2.3.2-3 | 5 | $1.2 \mathrm{D}+1.6 L_{r}-0.8 W$ | 1614.53 | -108.63 | 109.87 | -56.78 | 59.87 | -51.85 | 50.00 |

$\delta_{s}\left(M_{u, t o p}\right)_{s}=1.82 \times(-51.85)=-94.37 \mathrm{kN} . \mathrm{m}$
$\left(M_{u, t o p}\right)_{2^{n d}}=\left(M_{u, t o p}\right)_{n s}+\delta_{s}\left(M_{u, t o p}\right)_{s}=-56.78-94.37=-151.15 \mathrm{kN} . \mathrm{m}$
$\delta_{s}\left(M_{u, b o t}\right)_{s}=1.82 \times 50=91 \mathrm{kN} . \mathrm{m}$
$\left(M_{u, b o t}\right)_{2^{n d}}=\left(M_{u, b o t}\right)_{n s}+\delta_{s}\left(M_{u, b o t}\right)_{s}=59.87+91=150.87 \mathrm{kN} \cdot \mathrm{m}$

## Ex.1: 4. Moment Magnification at Ends of Compression Member

$$
\begin{aligned}
& \left(M_{2}\right)_{2^{n d}}=\max \left\{\left|\left(M_{u, t o p}\right)_{2^{n d}}\right|, \quad\left|\left(M_{u, b o t}\right)_{2^{n d}}\right|\right\} \Rightarrow\left(M_{u, t o p}\right)_{2^{n d}}=-151.15 \mathrm{kN} . \mathrm{m} \\
& \left(M_{2}\right)_{1^{s t}}=\left(M_{u, t o p}\right)_{1^{s t}}=-108.63 \mathrm{kN} . \mathrm{m} \\
& \frac{\left(M_{2}\right)_{2^{n d}}}{\left(M_{2}\right)_{1^{s t}}}=\frac{151.15}{108.63}=1.39 \leq 1.4 \mathrm{OK} \\
& \left(M_{1}\right)_{2^{n d}}=\min \left\{\left|\left(M_{u, t o p}\right)_{2^{n d}}\right|, \quad\left|\left(M_{u, b o t}\right)_{2^{n d}}\right|\right\} \Rightarrow\left(M_{u, b o t}\right)_{2^{n d}}=150.87 \mathrm{kN} \cdot \mathrm{~m} \\
& \left(M_{1}\right)_{1^{s t}}=\left(M_{u, b o t}\right)_{1^{s t}}=109.87 \mathrm{kN} \cdot \mathrm{~m} \\
& \frac{\left(M_{1}\right)_{2^{n d}}}{\left(M_{1}\right)_{1} s t}=\frac{150.87}{109.87}=1.37 \leq 1.4 \text { OK } \\
& P_{u}=1550.48 \mathrm{kN}
\end{aligned}
$$

Note: The designation of $M_{1}$ and $M_{2}$ is made based on the second-order (magnified) moments and not based on the first-order (unmagnified) moments.

## Ex.1: 5. Moment Magnification along Length of Compression Member

ACI 318-14 (6.6.4.6.4) : In sway frames, second-order effects shall be considered along the length of columns. It shall be permitted to account for these effects using ACI 318-14 (6.6.4.5) (Nonsway frame procedure), where $C_{m}$ is calculated using $M_{1}$ and $M_{2}$ from ACI 318-14 (6.6.4.6.1) as follows:

$$
M_{c}=\delta_{n s} M_{2}
$$

Where: $M_{2}=$ the second-order factored moment.

$$
\begin{gathered}
\delta_{n s}=\frac{C_{m}}{1-\frac{P_{u}}{0.75 P_{c}} \geq 1.0} \\
C_{m}=\left\{\begin{array}{ll}
0.6-0.4\left(\frac{M_{1}}{M_{2}}\right) & : \begin{array}{ll}
\text { من أجل الأعمدة من دون أحمال العرضية الأعمدة مع أحمال العرضية } & \text { من } \\
1 &
\end{array}
\end{array} .\right.
\end{gathered}
$$

## Ex.1: 5. Moment Magnification along Length of Compression Member

$$
\begin{aligned}
& P_{c}=\frac{\pi^{2}(E I)_{e f f}}{\left(k l_{u}\right)^{2}} \\
& (E I)_{e f f}= \begin{cases}(a) & \frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d n s}} \\
\text { (b) } & \frac{0.4 E_{c} I_{g}}{1+\beta_{d n s}} \\
\text { (c) } & \frac{E_{c} I}{1+\beta_{d n s}}\end{cases}
\end{aligned}
$$

/ from ACI 318M-14 (Table 6.6.3.1.1(b))
الحمل المحوري الأعظمي المصعد الكلي و المحمي المصعد المستديم منس تركيب الأحمال

There are three options for calculating the effective flexural stiffness of slender concrete columns (EI) eff $^{\text {The }}$ Thirst equation provides accurate representation of the reinforcement in the section and will be used in this example.

## Ex.1: 5. Moment Magnification along Length of Compression Member

$I_{g}=\frac{c^{4}}{12}=\frac{45^{4}}{12}=341718.75 \mathrm{~cm}^{4} \quad$ (Calculated previously)
$E_{c}=w_{c}^{1.5} 0.043 \sqrt{f_{c}^{\prime}}=2400^{1.5} \times 0.043 \times \sqrt{40}=31975.35 \mathrm{MPa} \quad$ (Calculated previously)
$\beta_{d n s}$ is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination

For load combination 5:

$$
\begin{aligned}
& P_{u, \text { sustained }}=1.2 D=1.2 \times 1258.85=1510.62 \mathrm{kN} \\
& P_{u}=1.2 D+1.6 L_{r}-0.8 \mathrm{~W}=1614.53 \mathrm{kN} \\
& \beta_{d n s}=\frac{P_{u, \text { sustained }}}{P_{u}}=\frac{1510.62}{1614.53}=0.94 \leq 1 \quad \text { OK }
\end{aligned}
$$

## Ex.1: 5. Moment Magnification along Length of Compression Member

With 8-\#20 reinforcement equally distributed on all sides and $45 \mathrm{~cm} \times 45 \mathrm{~cm}$ column section:
$I_{s e}=5772.68 \mathrm{~cm}^{4} \quad$ (Calculated previously)
$E I=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d n s}}$
$E I=\frac{0.2 \times 31975.35 \times 341718.75 \times 10^{4}+200000 \times 5772.68 \times 10^{4}}{1+0.97}$
$E I=16953.56 \times 10^{9} \mathrm{~N} . \mathrm{mm}^{2}$

## Ex.1: 5. Moment Magnification along Length of Compression Member

Effective Length Factor (A) Calculations for Exterior Columns with One Beam Framing into them in the Direction of Analysis (Nonsway Frame)
$\Psi_{A}=2.5 \quad$ (Calculated previously)
$\Psi_{B}=1$
Using Figure R6.2.5 from ACI 318M-14:
$k=0.83$ as shown in the figure for the exterior columns with one beam framing into them in the directions of analysis.
$P_{c}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 16953.56 \times 10^{9}}{(0.83 \times 4700)^{2}}$
$P_{c}=10995.34 \times 10^{3} \mathrm{~N}=10995.34 \mathrm{kN}$


عوامل الطول الفعال الغير منزاحة اللعانبياً

## Ex.1: 5. Moment Magnification along Length of Compression Member

For load combination 5:
$P_{u}=1.2 D+1.6 L_{r}-0.8 W=1614.53 \mathrm{kN}$
$C_{m}=0.6-0.4\left(\frac{M_{1}}{M_{2}}\right)$
$M_{2}=\left(M_{2}\right)_{2}$ nd $=-151.15 \mathrm{kN} . \mathrm{m} \quad$ (as concluded from section 4)
$M_{1}=\left(M_{1}\right)_{2^{n d}}=150.87 \mathrm{kN} . \mathrm{m} \quad$ (as concluded from section 4)
Since the column is bent in double curvature, $M_{1} / M_{2}$ is positive.
$C_{m}=0.6-0.4\left(\frac{150.87}{151.15}\right)=0.201$
$\delta_{n s}=\frac{C_{m}}{1-\frac{P_{u}}{0.75 P_{c}}}=\frac{0.201}{1-\frac{1614.53}{0.75 \times 10995.34}}=0.25<1.0 \Rightarrow \delta_{n s}=1$


## Ex.1: 5. Moment Magnification along Length of Compression Member

$$
\begin{aligned}
& M_{\min }=P_{u}(15+0.03 \mathrm{~h}) \\
& P_{u}=1614.53 \mathrm{kN}, \text { and } h=450 \mathrm{~mm} \text { the section dimension in the direction being considered. } \\
& M_{\min }=P_{u}(15+0.03 \mathrm{~h})=1614.53(0.015+0.03 \times 0.45)=46.01 \mathrm{kN} . \mathrm{m} \\
& \left|M_{2}\right|=151.15 \mathrm{kN} . \mathrm{m}>M_{\min }=46.01 \mathrm{kN} . \mathrm{m} \Rightarrow M_{2}=-151.15 \mathrm{kN} . \mathrm{m} \\
& M_{1}=150.87 \mathrm{kN} . \mathrm{m}>M_{\min }=46.01 \mathrm{kN} . \mathrm{m} \Rightarrow M_{1}=150.87 \mathrm{kN} . \mathrm{m} \\
& M_{c 2}=\delta_{n s} M_{2}=1 \times(-151.15)=-151.15 \mathrm{kN} . \mathrm{m} \\
& M_{c 1}=\delta_{n s} M_{1}=1 \times 150.87=150.87 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

## Ex.1: 5. Moment Magnification along Length of Compression Member

$M_{c 1}$ and $M_{c 2}$ will be considered separately to ensure proper comparison of resulting magnified moments against negative and positive moment capacities of unsymmetrical sections as can be seen in the following figure.


## Ex.2: Design of a slender column in a nonsway frame.

Figure below shows an elevation view of a multistory concrete frame building, with 120 cm wide $\times 30 \mathrm{~cm}$ deep beams on all column lines, carrying two-way slab floors and roof. The clear height of the columns is 3.95 m Interior columns are tentatively dimensioned at $45 \times 45 \mathrm{~cm}$, and exterior columns at $40 \times 40 \mathrm{~cm}$. The frame is effectively braced against sway by stair and elevator shafts having concrete walls that are monolithic with the floors, located in the building corners (not shown in the figure).


## Ex.2: Design of a slender column in a nonsway frame.

The structure will be subjected to vertical dead and live loads. Trial calculations by firstorder analysis indicate that the pattern of live loading shown in Figure, with full load distribution on roof and upper floors and a checkerboard pattern adjacent to column C3, produces maximum moments with single curvature in that column, at nearly maximum axial load. Dead loads act on all spans. Service load values of dead and live load axial force and moments for the typical interior column C3 are as follows:

| Dead load | Live load |
| :---: | :---: |
| $P=990 \mathrm{kN}$ | $\mathrm{P}=745 \mathrm{kN}$ |
| $M_{\text {top }}=30 \mathrm{kN} . \mathrm{m}$ | $M_{\text {top }}=126 \mathrm{kN} . \mathrm{m}$ |
| $M_{\text {bot }}=-30 \mathrm{kN} . \mathrm{m}$ | $M_{\text {bot }}=147.5 \mathrm{kN} . \mathrm{m}$ |

ملاحظة 1: إشارة العزم تدل على جهة شد الألياف و ليس على جهة الدوران.
ملاحظة 2: الإشارة السالبة للقوة المحورية تعني الشد، بينما الموجبة تعني الضغط.
The column is subjected to double curvature under dead load alone and single curvature under live load.

Design column C3, using the ACI moment magnifier method. Use $f_{c}^{\prime}=28 \mathrm{MPa}$ psi and $f_{y}=420 \mathrm{MPa}$.

## Ex.2: Solution

The column will first be designed as a short column, assuming no slenderness effect. With the application of the usual load factors,

$$
\begin{aligned}
& P_{u}=1.2 D+1.6 L=1.2 \times 990+1.6 \times 745=2380 \mathrm{kN} \\
& M_{u}=1.2 D+1.6 L=\max \left\{\begin{array}{c}
1.2 \times 30+1.6 \times 126=237.6 \mathrm{kN} . \mathrm{m} \\
1.2 \times(-30)+1.6 \times 147.5=200 \mathrm{kN} . \mathrm{m}
\end{array}\right.
\end{aligned}
$$

For a $45 \times 45 \mathrm{~cm}$ column, with the 4 cm clear to the outside steel, No. 10 stirrups, and (assumed) No. 30 longitudinal steel:

$$
\gamma=(450-2 \times 40-2 \times 10-30) / 450=0.71
$$

Graph A for $\gamma=0.70$, with bars arranged around the column perimeter, will be used. Then

$$
\begin{aligned}
& \frac{P_{u}}{\phi f_{c}^{\prime} A_{g}}=\frac{2380 \times 10^{3}}{0.65 \times 28 \times 202500}=0.646 \\
& \frac{M_{u}}{\phi f_{c}^{\prime} A_{g} h}=\frac{237.6 \times 10^{6}}{0.65 \times 28 \times 202500 \times 450}=0.143
\end{aligned}
$$



## Ex.2: Solution

$$
\begin{aligned}
& R_{n}=\frac{M_{u}}{\phi f_{c}^{\prime} A_{g} h}=0.143 \\
& K_{n}=\frac{P_{u}}{\phi f_{c}^{\prime} A_{g}}=0.646
\end{aligned}
$$


and from the graph $\rho_{g}=0.02$. This is low enough that an increase in steel area could be made, if necessary, to allow for slenderness, and the $45 \times 45 \mathrm{~cm}$ concrete dimensions will be retained.

$$
\text { Graph A } \quad R_{n}=\frac{P_{n} e}{f_{c}^{\prime} A_{g} h}=\frac{P_{u} e}{\phi f_{c}^{\prime} A_{g} h}
$$

## Ex.2: Solution

For an initial check on slenderness, an effective length factor $k=1.0$ will be used. Then

$$
\begin{aligned}
& \frac{k l_{u}}{r}=\frac{1.0 \times 3950}{0.3 \times 450}=29.3 \\
& M_{1}=M_{\mathrm{bot}}=1.2 \times(-30)+1.6 \times 147.5=200 \mathrm{kN} . \mathrm{m} \\
& M_{2}=M_{\mathrm{top}}=1.2 \times 30+1.6 \times 126=237.6 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

For a braced frame, the upper limit for short column behavior is

$$
\min \left\{\begin{array}{l}
34+12\left(\frac{M_{1}}{M_{2}}\right)=34+12\left(-\frac{200}{237.6}\right)=23.9 \\
40
\end{array}\right\}=23.9
$$

النسبة $M_{1} / M_{2}$ سالبة عندما يكون العمود منحنياً باتجاه واحد
The calculated value of 29.3 exceeds this, so slenderness must be considered in the design. A more refined calculation of the effective length factor $k$ is thus called for.

## Ex.2: Solution

Because $E_{c}$ is the same for column and beams, it will be canceled in the stiffness calculations.
$I_{c}=0.7 \frac{c^{4}}{12}=0.7 \times \frac{45^{4}}{12}=239203.13 \mathrm{~cm}^{4}$
giving $\frac{I_{c}}{l_{c}}=\frac{239203.13}{425}=562.83 \mathrm{~cm}^{3}$
For the beams, the moment of inertia will be taken as $0.35 /_{g}$, where $I_{g}$ is taken as 2 times the gross moment of inertia of the web. Thus,
$I_{b}=0.35 \times 2 \times \frac{b_{w} h_{b}^{3}}{12}=0.35 \times 2 \times \frac{120 \times 30^{3}}{12}=189000 \mathrm{~cm}^{4}$
$\frac{I_{b}}{l_{b}}=\frac{189000}{730}=258.90 \mathrm{~cm}^{3}$
$\Psi_{A}=\Psi_{B}=\frac{\sum E_{c} I_{c} / L_{c}}{\sum E_{b} I_{b} / L_{b}}=\frac{2 \times 562.83}{2 \times 258.90}=2.17$

## Ex.2: Solution

$\Psi_{A}=\Psi_{B}=2.17$

From Figure aside for the braced frame, the value of $k$ is 0.87 , rather than 1.0 as used previously. Consequently,
$\frac{k l_{u}}{r}=\frac{0.87 \times 3950}{0.3 \times 450}=25.5>23.9$

This is still above the limit value of 23.9 confirming that slenderness must be considered.


عوامل الطول الفعال للعناصر المضغوٍ
في الإطارات الغير منزاحة جانبياً

## Ex.2: Solution

A check will now be made of minimum moment.
$M_{\text {min }}=P_{u}(15+0.03 h)$
$P_{u}=2380 \mathrm{kN}$, and $h=45 \mathrm{~cm}$ the section dimension in the direction being considered.
$M_{\text {min }}=P_{u}(15+0.03 \mathrm{~h})=2380(0.015+0.03 \times 0.45)=67.8 \mathrm{kN} . \mathrm{m}<M_{u}=237.6 \mathrm{kN} . \mathrm{m}$
It is seen that this does not control.
The coefficient $C_{m}$ can now be found with
$M_{1}=M_{b o t}=1.2 D+1.6 L=1.2 \times(-30)+1.6 \times 147.5=200 \mathrm{kN} . \mathrm{m}$
$M_{2}=M_{\text {top }}=1.2 D+1.6 L=1.2 \times 30+1.6 \times 126=237.6 \mathrm{kN} . \mathrm{m}$
Since the column is bent in double curvature, $M_{1} / M_{2}$ is negative.
$C_{m}=0.6-0.4\left(\frac{M_{1}}{M_{2}}\right)=0.6+0.4\left(\frac{200}{237.6}\right)=0.937$


## Ex.2: Solution

Next the factor $\beta_{d n s}$ will be found based on the ratio of the maximum factored axial sustained load (the factored dead load in this case) to the maximum factored axial load:

$$
\beta_{d n s}=\frac{P_{u, \text { sustained }}}{P_{u}}=\frac{1.2 \times 990}{1.2 \times 990+1.6 \times 745}=0.50 \leq 1 \quad \text { OK }
$$

For a relatively low reinforcement ratio, one estimated to be in the range of 0.02 to 0.03 , the more approximate equation for $E /$ will be used:

$$
\begin{aligned}
(E I)_{e f f} & =\frac{0.4 E_{c} I_{g}}{1+\beta_{d n s}}=\frac{0.4 \times 4700 \sqrt{28} \times 450^{4} / 12}{1+0.50}=2.266 \times 10^{13} \mathrm{~N} . \mathrm{mm}^{2} \\
& =2.266 \times 10^{8} \mathrm{kN.} \mathrm{~cm}
\end{aligned}
$$

The critical buckling load is found from

$$
P_{c}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 2.266 \times 10^{8}}{(0.87 \times 395)^{2}}=1.894 \times 10^{4} \mathrm{kN}
$$

## Ex.2: Solution

The moment magnification factor can now be found from

$$
\begin{equation*}
\delta_{n s}=\frac{C_{m}}{1-\frac{P_{u}}{0.75 P_{c}}}=\frac{0.937}{1-\frac{2380}{0.75 \times 18940}}=1.126>1 \tag{OK}
\end{equation*}
$$

Thus, the required axial strength of the column is $P_{u}=2380 \mathrm{kN}$ (as before), while the magnified design moment is:

$$
M_{c}=\delta_{n s} M_{2}=1.126 \times 237=266.86 \mathrm{kN} . \mathrm{m}
$$

ACI Code 6.2.6 limits the magnified moment to 1.4 times the moment due to first-order effects. This limitation is clearly satisfied.

With reference again to the column design chart A with

$$
\begin{aligned}
& \frac{P_{u}}{\phi f_{c}^{\prime} A_{g}}=\frac{2380 \times 10^{3}}{0.65 \times 28 \times 202500}=0.646 \\
& \frac{M_{u}}{\phi f_{c}^{\prime} A_{g} h}=\frac{266.86 \times 10^{6}}{0.65 \times 28 \times 202500 \times 450}=0.161
\end{aligned}
$$

## Ex.2: Solution

$$
\begin{aligned}
& R_{n}=\frac{M_{u}}{\phi f_{c}^{\prime} A_{g} h}=0.161 \\
& K_{n}=\frac{P_{u}}{\phi f_{c}^{\prime} A_{g}}=0.646
\end{aligned}
$$



عوامل الطول الفعال للعناصر المضغوطة في الإطارات الغير منزاحة جانبياً

## Ex.2: Solution

it is seen that the required reinforcement ratio is increased from 0.020 to 0.026 because of slenderness. The steel area now required is

$$
A_{s t}=0.027 \times 202500=5468 \mathrm{~mm}^{2}
$$

which can be provided using eight No. 30 bars ( $A_{s t}=5655 \mathrm{~mm}^{2}$ ), arranged as shown in figure below. No. 10 ties will be used at a spacing not to exceed the least dimension of the column ( 450 mm ), 48 tie diameters ( 480 mm ), or 16 bar diameters ( 480 mm ). Single ties at 450 mm spacing, as shown in the figure, will meet requirements of the ACl Code.


## Ex.2: Solution

Further refinements in the design could, of course, be made by recalculating the critical buckling load using Equation shown below. This extra step is not justified here because the column slenderness is barely above the upper limit for short column behavior and the moment magnification is not great.

$$
(E I)_{e f f}=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d n s}}
$$

## Ex.3: Design of a slender column in a sway frame.

Consider now that the concrete building frame of Example 2 acts as a sway frame, without the stairwells or elevator shafts described earlier. An initial evaluation is carried out using the member dimensions and reinforcement given in Example 2. The reinforcement for the interior $45 \times 45 \mathrm{~cm}$ columns consists of eight No. 30 bars. Reinforcement for the exterior $40 \times 40 \mathrm{~cm}$ columns consists of eight No. 25 bars.


## Ex.3: Design of a slender column in a sway frame.



## Ex.3: Design of a slender column in a sway frame.

The building will be subjected to gravity dead and live loads and horizontal wind loads. Elastic first-order analysis of the frame at service loads (all load factors = 1.0) gives the following results at the third story:

|  | Cols. A3 and F3 | Cols. B3 and E3 | Cols. C3 and D3 |
| :--- | ---: | ---: | ---: |
| $P_{\text {dead }}$ | 496 kN | 990 kN | 990 kN |
| $P_{\text {live }}$ | 388 kN | 745 kN | 745 kN |
| $P_{\text {wind }}$ | $\pm 129 \mathrm{kN}$ | $\pm 78 \mathrm{kN}$ | $\pm 26 \mathrm{kN}$ |
| $V_{\text {wind }}$ | 24 kN | 48 kN | 48 kN |
| $M_{\text {top, dead }}$ |  |  | $30 \mathrm{kN.m}$ |
| $M_{\text {top, live }}$ |  |  | $126 \mathrm{kN.m}$ |
| $M_{\text {top, wind }}$ |  |  | $\pm 102 \mathrm{kN.m}$ |
| $M_{\text {bot, dead }}$ |  |  | $-30 \mathrm{kN.m}$ |
| $M_{\text {bot, live }}$ |  |  | $147.5 \mathrm{kN.m}$ |
| $M_{\text {bot, wind }}$ |  |  | $\pm 92 \mathrm{kN.m}$ |

## Ex.3: Design of a slender column in a sway frame.

To simplify the analysis in this example, roof loads will not be considered. The relative lateral deflection for the third story under total wind shear $V_{\text {wind }}=240 \mathrm{kN}$ is 20 mm .

Column C3 is to be designed for the critical loading condition, using the ACI moment magnifier method.
Use $f_{c}^{\prime}=28 \mathrm{MPa}$ psi and $f_{y}=420 \mathrm{MPa}$.

## Ex.3: Solution

Initially, a check is made to see if a sway frame analysis is required.
The following load combinations provide the greatest value of $\sum P_{u}$

$$
1.2 D+1.6 W+1.0 L+0.5\left(L_{r} \text { or } S \text { or } R\right)
$$

The factored shear

$$
V_{u s}=1.6 V_{\text {wind }}=1.6 \times 240=384 \mathrm{kN}
$$

The corresponding deflection

$$
\Delta_{0}=1.6 \Delta=1.6 \times 20=32 \mathrm{~mm}
$$

The total factored axial force on the story is obtained using the load table.
Columns A3 and F3: $\quad P_{u}=1.2 D+1.0 L=1.2 \times 496+1.0 \times 388=983 \mathrm{kN}$

Columns B3, C3, D3 and E3: $\quad P_{u}=1.2 D+1.0 L=1.2 \times 990+1.0 \times 745=1933 \mathrm{kN}$

## Ex.3: Solution

Note that in this case the values of $P_{\text {wind }}$ in the columns are not considered since they cancel out for the floor as a whole, that is,
$\sum P_{\text {wind }}=0$
Thus,
$\sum P_{u}=2 \times 983+4 \times 1933=9698 \mathrm{kN}$
and the stability index is
$Q=\frac{\left(\sum P_{u}\right) \Delta_{0}}{V_{u s} l_{c}}=\frac{9698 \times 32}{384 \times 4250}=0.19>0.05$
Since $Q>0.05$, sway frame analysis is required for this story.

## Ex.3: Solution

(a) Gravity loads only.

All columns in sway frames must first be considered as braced columns under gravity loads acting alone, that is, for

$$
U=1.2 D+1.6 L
$$

This check has already been made for column C3 in Example 2

## Ex.3: Solution

(b) Gravity plus wind loads.

Three additional load combinations must be considered when wind effects are included:

$$
\begin{aligned}
& W 1=1.2 D+1.6\left(L_{r} \text { or } S \text { or } R\right) \pm 0.8 W \\
& W 2=1.2 D+f_{1} L+0.5\left(L_{r} \text { or } S \text { or } R\right) \pm 1.6 W \\
& W 3=0.9 D \pm 1.6 W
\end{aligned}
$$

تركيب الرياح الأول المتحكم بالضغط:
تركيب الرياح الثاني المتحكم بالضغط:
تركيب الرياح المتحكم بالشد:
f الأحمال الحية 5KN/m² وفي الأحمال الحية لمرائب السيارات، ويساوي 0.5 لباقي الأحمال الحية.

By inspection, the second combination will control for this case, and the others will not be considered further.

## Ex.3: Solution

From Example 2, $\psi_{A}=\psi_{B}=2.17$. With reference to the alignment chart, the effective length factor for an unbraced frame $\mathrm{k}=1.64$ and

$$
\frac{k l_{u}}{r}=\frac{1.64 \times 3950}{0.3 \times 450}=48.0>22
$$

This is much above the limit value of 22 for short column behavior in an unbraced frame. (This should be no surprise since $k /_{u} / r=25.5$ for column C3 in the braced condition.)

## Ex.3: Solution

For sway frame analysis, the loads must be separated into gravity loads and sway loads, and the appropriate magnification factor must be applied to the sway moments. The factored end moments resulting from the nonsway loads on column C3 are

$$
\begin{aligned}
& \left(M_{u, t o p}\right)_{n s}=1.2 \times 30+1.0 \times 126=162 \mathrm{kN} . \mathrm{m} \\
& \left(M_{u, b o t}\right)_{n s}=1.2 \times(-30)+1.0 \times 147.5=111.5 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

The sway effects will amplify the moments:

$$
\begin{aligned}
& \left(M_{u, t o p}\right)_{s}=1.6 \times 102=163 \mathrm{kN} \cdot \mathrm{~m} \\
& \left(M_{u, b o t}\right)_{s}=1.6 \times(-92)=-147 \mathrm{kN} \cdot \mathrm{~m} \\
& \left(M_{u, t o p}\right)_{1} s t=\left(M_{u, t o p}\right)_{n s}+\left(M_{u, t o p}\right)_{s}=162+163=325 \mathrm{kN} \cdot \mathrm{~m} \\
& \left(M_{u, b o t}\right)_{1} s t=\left(M_{u, b o t}\right)_{n s}+\left(M_{u, b o t}\right)_{s}=111.5-147=-35.5 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

## Ex.3: Solution

For the purposes of comparison, the magnified sway moments will be calculated based on both:

$$
\delta_{s} M_{s}=\frac{M_{s}}{1-Q} \quad \delta_{s} M_{s}=\frac{M_{s}}{1-\frac{\sum P_{u}}{0.75 \sum P_{c}}}
$$

Using the first equation

$$
\delta_{s} M_{s}=\frac{M_{s}}{1-Q}=\frac{1}{1-0.19}=1.23
$$

giving

$$
\begin{aligned}
& \delta_{s}\left(M_{u, t o p}\right)_{s}=1.23 \times 163=200 \mathrm{kN} . \mathrm{m} \\
& \delta_{s}\left(M_{u, b o t}\right)_{s}=1.23 \times(-147)=-181 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

## Ex.3: Solution

To use the second equation, the critical loads must be calculated for each of the columns For columns A3 and F3,

Columns:
$I_{c}=0.7 \frac{c^{4}}{12}=0.7 \times \frac{40^{4}}{12}=149333.33 \mathrm{~cm}^{4}$
$\frac{I_{c}}{l_{c}}=\frac{149333.33}{425}=351.37 \mathrm{~cm}^{3}$
Beams:
$I_{b}=0.35 \times 2 \times \frac{b_{w} h_{b}^{3}}{12}=0.35 \times 2 \times \frac{120 \times 30^{3}}{12}=189000 \mathrm{~cm}^{4}$
$\frac{I_{b}}{l_{b}}=\frac{189000}{730}=258.90 \mathrm{~cm}^{3}$
Rotational restraint factors for this case, with two columns and one beam framing into the joint, are
$\Psi_{A}=\Psi_{B}=\frac{\sum E_{c} I_{c} / L_{c}}{\sum E_{b} I_{b} / L_{b}}=\frac{2 \times 351.37}{258.90}=2.71$

## Ex.3: Solution

which, with reference to the alignment chart for unbraced frames, gives $\mathrm{k}=1.77$.

For wind load, $\beta_{d s}=0$.
Since reinforcement has been initially selected for one column, El will be calculated using:
$(E I)_{e f f}=\frac{0.2 E_{c} I_{g}+E_{s} I_{s e}}{1+\beta_{d s}}$
$(E I)_{e f f}=0.2 \times 4700 \sqrt{28} \times \frac{400^{4}}{12}+200000 \times 6 \times \frac{\pi 25^{2}}{4} \times(200-40-10-12.5)^{2}$
$(E I)_{e f f}=2.175 \times 10^{13} \mathrm{~N} . \mathrm{mm}^{2}$
$(E I)_{e f f}=2.175 \times 10^{8} \mathrm{kN.cm}{ }^{2}$
The critical buckling load is found from
$P_{c}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 2.175 \times 10^{8}}{(1.77 \times 395)^{2}}=4391.55 \mathrm{kN}$


Exterior Columns

## Ex.3: Solution

For columns B3, C3, D3, and E3, from earlier calculations for column C3, k=1.64 for the sway loading case. For these columns,
$(E I)_{e f f}=0.2 \times 4700 \sqrt{28} \times \frac{450^{4}}{12}+200000 \times 6 \times \frac{\pi 30^{2}}{4} \times(225-40-10-15)^{2}$
$(E I)_{e f f}=3.871 \times 10^{13} \mathrm{~N} . \mathrm{mm}^{2}$
$(E I)_{e f f}=3.871 \times 10^{8} \mathrm{kN.cm}{ }^{2}$

The critical buckling load is found from

$$
P_{c}=\frac{\pi^{2} E I}{\left(k l_{u}\right)^{2}}=\frac{\pi^{2} \times 3.871 \times 10^{8}}{(1.64 \times 395)^{2}}=9104.18 \mathrm{kN}
$$

Clear cover 40 mm
Interior Columns


## Ex.3: Solution

Thus, for all the columns at this level of the structure,

$$
\begin{aligned}
& \sum P_{c}=2 \times 4391.55+4 \times 9104.18 \Rightarrow \sum P_{c}=45199.82 \mathrm{kN} \\
& \sum P_{u}=9698 \mathrm{kN} \\
& \delta_{s}=\frac{1}{1-\frac{\sum P_{u}}{0.75 \sum P_{c}}} \geq 1 \Rightarrow \quad \delta_{s}=\frac{1}{1-\frac{9698}{0.75 \times 45199.82}}=1.4
\end{aligned}
$$

and the magnified sway moments for the top and bottom of column C3 are

$$
\begin{aligned}
& \delta_{s}\left(M_{u, t o p}\right)_{s}=1.4 \times 163=228.2 \mathrm{kN} . \mathrm{m} \\
& \delta_{s}\left(M_{u, b o t}\right)_{s}=1.4 \times(-147)=-205.8 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

The values of $\delta \mathrm{s}$ Ms are higher based on $\Sigma \mathrm{Pu} / \Sigma \mathrm{Pc}$ than they are based on $\mathrm{Q}(228.2 \mathrm{kN} . \mathrm{m}$ vs. $200 \mathrm{kN} . \mathrm{m}$ for $\delta \mathrm{s} \mathrm{M} 2 \mathrm{~s}$ ), emphasizing the conservative nature of the moment magnifier approach based on $\Sigma \mathrm{Pu} / \mathrm{LPc}$.

## Ex.3: Solution

The design will proceed using the less conservative value of $\delta s M s$. The total magnified moments are

$$
\begin{aligned}
& \left(M_{u, t o p}\right)_{2} n d=\left(M_{u, t o p}\right)_{n s}+\delta_{s}\left(M_{u, t o p}\right)_{s}=162+200=362 \mathrm{kN} . \mathrm{m} \\
& \left(M_{u, b o t}\right)_{2} n d=\left(M_{u, b o t}\right)_{n s}+\delta_{s}\left(M_{u, b o t}\right)_{s}=111.5-181=-69.5 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\left(M_{u, t o p}\right)_{2^{n d}}}{\left(M_{u, t o p}\right)_{1^{s t}}}=\frac{350}{325}=1.08 \leq 1.4 \quad \mathrm{OK} \\
& \frac{\left(M_{u, b o t}\right)_{2^{n d}}}{\left(M_{u, b o t}\right)_{1^{s t}}}=\frac{-69.5}{-35.5}=1.96 \pm 1.4 \quad \text { Not OK }
\end{aligned}
$$

وبالتالي فإنه يجب تعديل أبعاد المقطع أو الجملة الإنشائية.

