نقابـــة المهندســــــين

الدورة التأهيلية لنيل مرتبة المهندس الاستشاري

في اختصاص هندسة الجيوتكنيك

Lecture

1

Compressibility of Soil

انضغاطـــية التربــــة

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References

Das, B., M. (2014), " Principles of geotechnical Engineering " Eighth Edition, CENGAGE Learning, ISBN-13: 978-0-495-41130-7.

Das, B., M. (2012), " Principles of Foundation Engineering " Eighth Edition, CENGAGE Learning, ISBN-13: 978-1-305-08155-0.

Introduction

Structures are built on soils. They transfer loads to the subsoil through the foundations. The effect of the loads is felt by the soil normally up to a depth of about four times the width of the foundation. The soil within this depth gets compressed due to the imposed stresses. The compression of the soil mass leads to the decrease in the volume of the mass which results in the settlement of the structure.

Introduction

If the settlement is not kept to tolerable limit, the desire use of the structure may be impaired and the design life of the structure may be reduced

It is therefore important to have a mean of predicting the amount of soil compression or consolidation

Compressibility

The settlement is defined as the compression of a soil layer due to the loading applied at or near its top surface.

The total compression of soil under load is composed of three components (i.e. elastic settlement, primary consolidation settlement, and secondary compression).

Compressibility

There are three types of settlement:

1. Immediate or Elastic Settlement (Se): caused by the elastic deformation of dry soil and of moist and saturated soils without change in the moisture content.
2. Primary Consolidation Settlement (Sc): volume change in saturated cohesive soils as a result of expulsion of the water that occupies the void spaces.

Compressibility

3.Secondary Consolidation Settlement (Ss):
volume change due to the plastic adjustment of soil fabrics under a constant effective stress (creep).

Coarse-grained soils do not undergo consolidation settlement due to relatively high hydraulic conductivity compared to clayey soils. Instead, coarse-grained soils undergo immediate settlement.

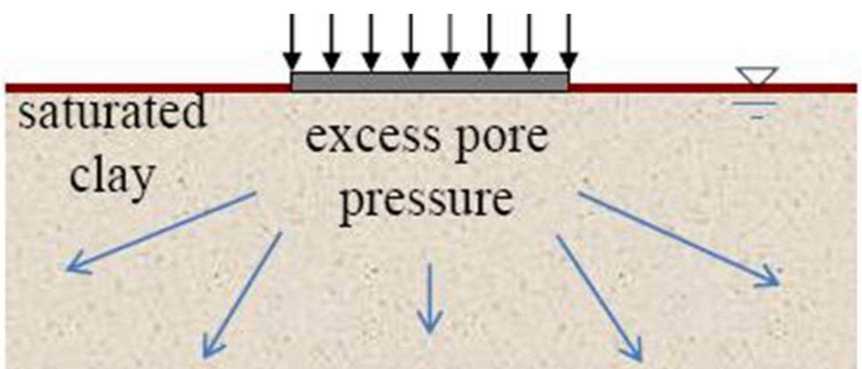
Consolidation

Consolidation settlement is the vertical displacement of the surface corresponding to the volume change in saturated cohesive soils as a result of expulsion of the water that occupies the void spaces.

- *Consolidation settlement will result, for example, if a structure is built over a layer of saturated clay or if the water table is lowered permanently in a stratum overlying a clay layer.*

Consolidation

Consolidation is the time-dependent settlement of fine grained soils resulting from the expulsion of water from the soil pores. The rate of escape of water depends on the permeability of the soil.



Consolidation

- *Permeability of clay is low*
- *Drainage occurs slowly – therefore, the settlement is delayed.*
- *Clayey soils undergo consolidation settlement not only under the action of "external" loads (surcharge loads) but also under its own weight or weight of soils that exist above the clay (geostatic loads).*
- *Clayey soils also undergo settlement when dewatered (e.g., ground water pumping) – because the effective stress on the clay increases.*

Consolidation

The amount of settlement is proportional to the one-dimensional strain caused by variation in the effective stress. The rate of settlement is a function of the soil type, the geometry of the profile (in 1-D consolidation, the length of the drainage path) and a mathematical solution between a time factor and the percent consolidation which has occurred.

Consolidation vs. Compaction

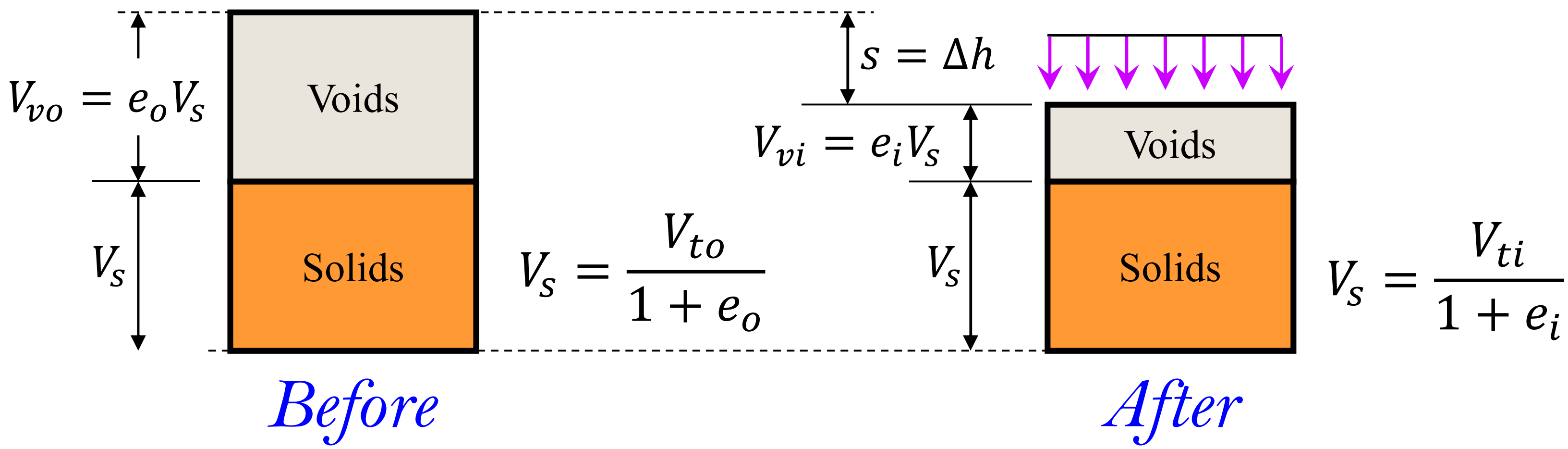
Consolidation vs. Compaction

Compaction	Consolidation
Instantaneous Process (applicable to all soils)	*Time-dependent process (applicable to clayey soils Only). Can occur over 100s of years*
Applicable to unsaturated soils. Decrease in air voids (not water voids).	*Applicable to saturated soils. Decrease in water voids (air voids do not exist)*
Dry density increases, water content dose not change	*Dry density increases, water content decreases.*

Consolidation

Consolidation settlement

The clay layer is shown as a phase diagram.

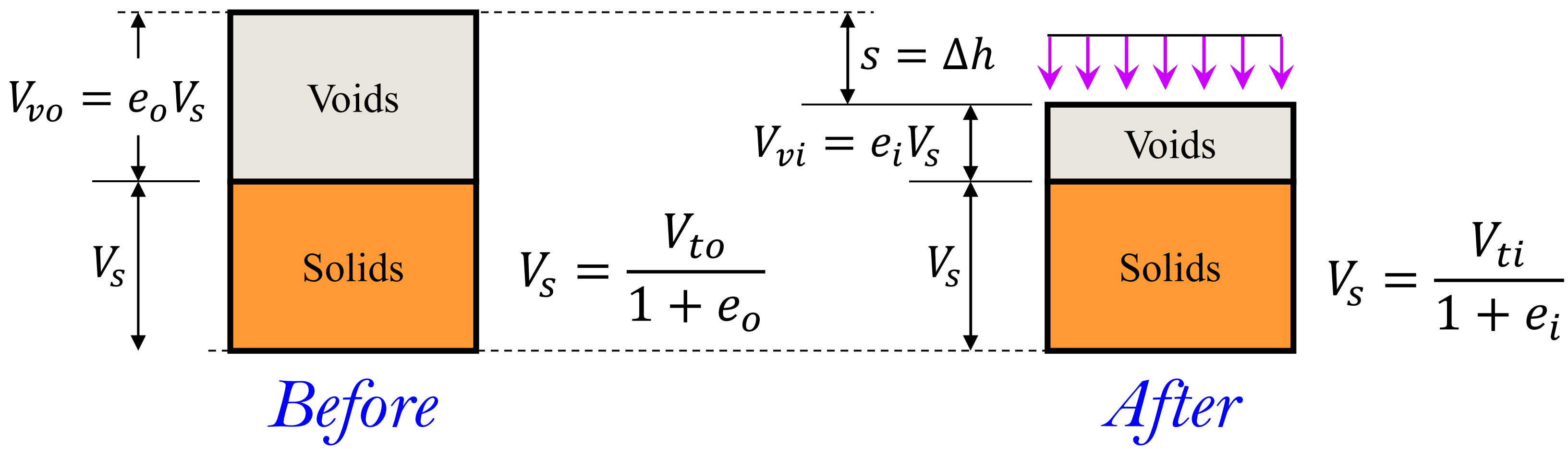


The volume of solid remains constant in the soil profile. Any change in height in the soil is equal to the change in height of voids.

Consolidation

Consolidation settlement

The clay layer is shown as a phase diagram.



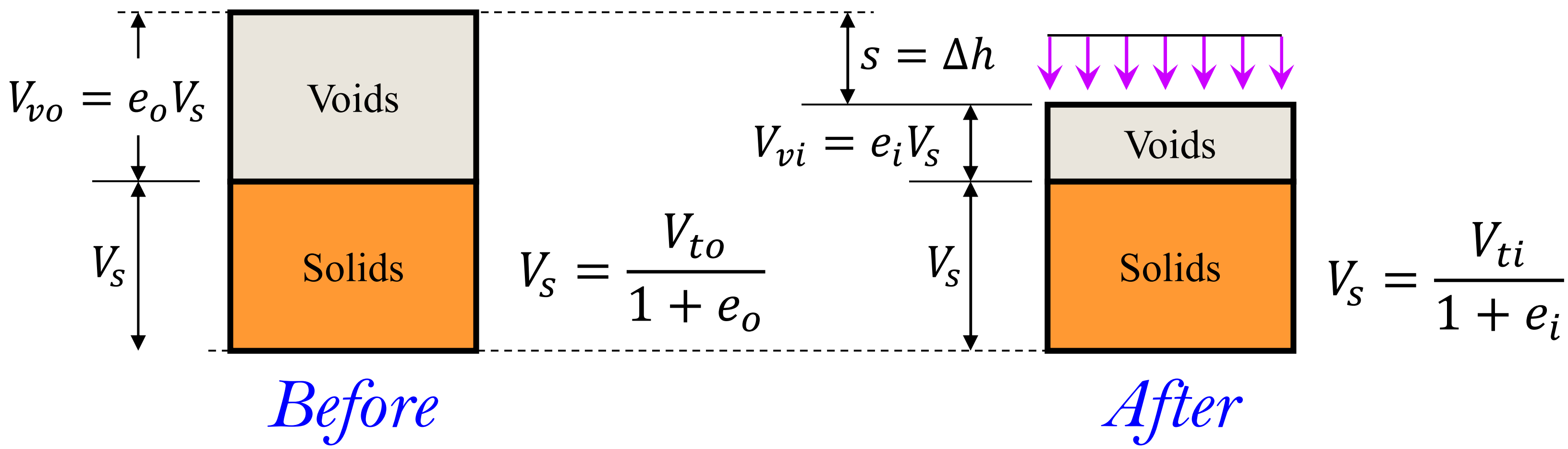
If the total and void volumes are divided by a unit cross-sectional area, the respective heights are determined.

$$\frac{V_{ti}}{V_{to}} = \frac{1+e_i}{1+e_o} \rightarrow \frac{h_i}{h} = \frac{1+e_i}{1+e_o} \rightarrow \frac{h-\Delta h}{h} = \frac{1+e_i}{1+e_o}$$

Consolidation

Consolidation settlement

The clay layer is shown as a phase diagram.



The strain that occurs in the clay is equal to the change in height divided by the original height, or

$$\varepsilon = \frac{\Delta h}{h} = \frac{\Delta e}{1 + e_o} \qquad e_i = e_o - (1 + e_o)\frac{\Delta h}{h}$$

Note, h is the full height of clay layer.

Fundamentals of Consolidation

Spring-cylinder model

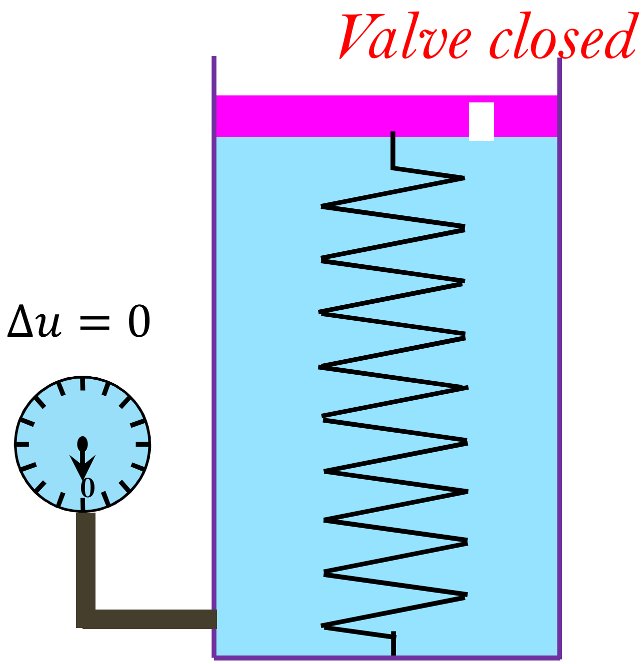
During consolidation , $\Delta\sigma$ remains the same, Δu decreases (due to drainage) while $\Delta\sigma'$ increases, transferring the load from water to the soil.

Fundamentals of Consolidation

Spring-cylinder model

The time-dependent deformation of saturated clayey soil can best understood by considering a simple model that consist of a cylinder with a spring at its center.

The cylinder is filled with water and has a frictionless watertight piston and valve.



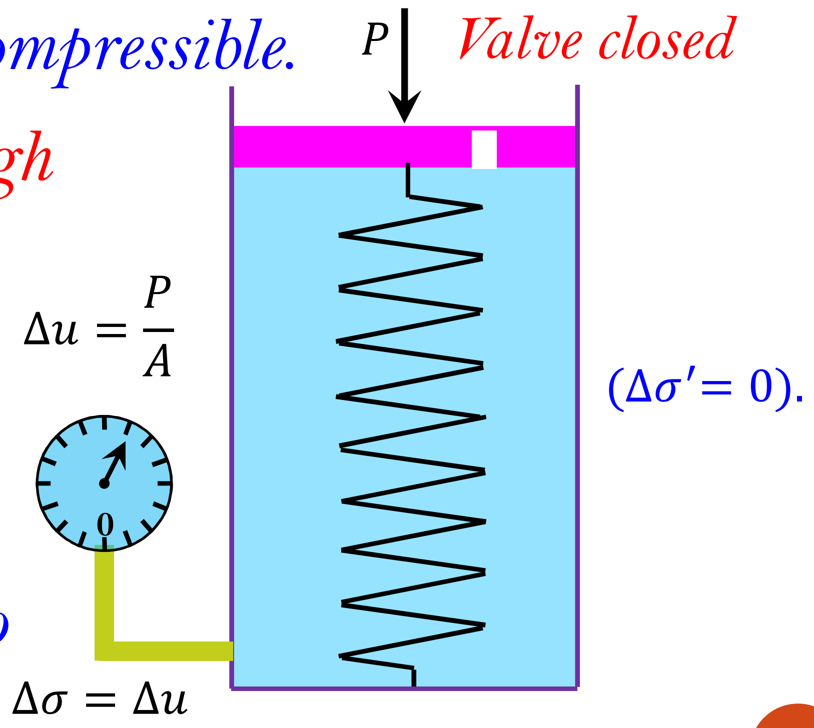
Fundamentals of Consolidation

Spring-cylinder model

If we place a load P *on the piston and keep the valve closed. The entire load will be taken by the water in the cylinder because water is incompressible.*

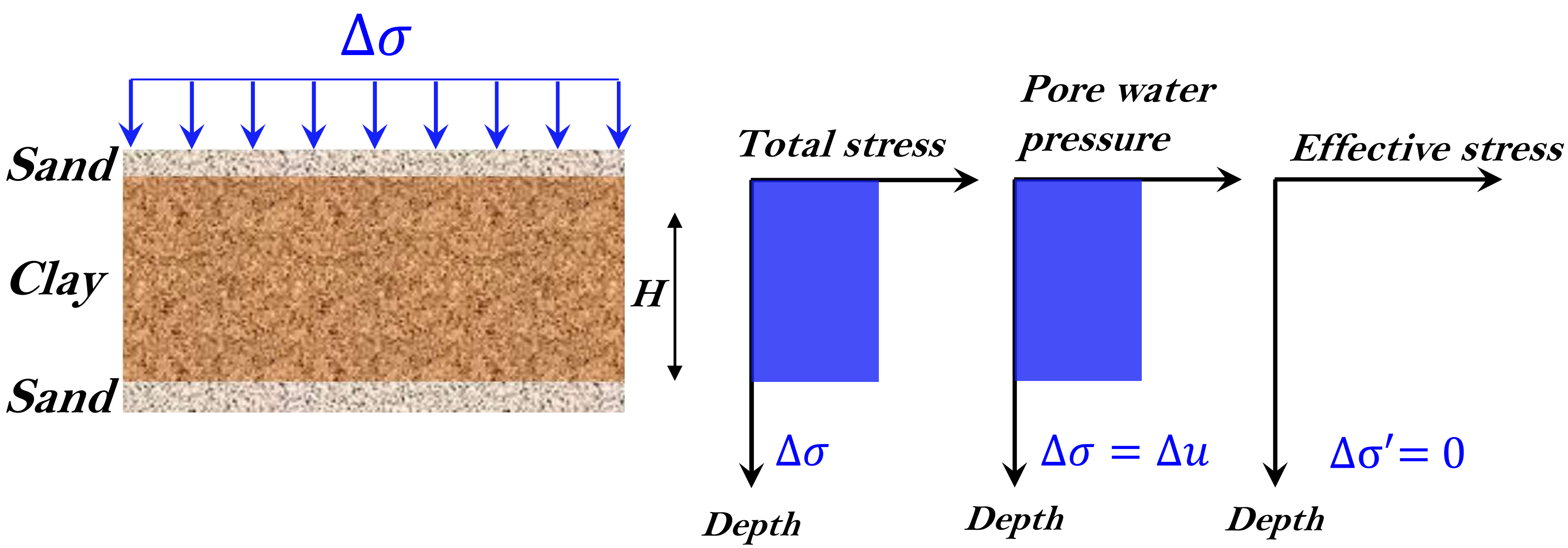
The spring will not go through any deformation.

The excess hydrostatic pressure at this time can be given as $\Delta u = P/A$ *and the effective stress is equal to zero*



Fundamentals of Consolidation

Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress $\Delta\sigma$:



At time = 0

Fundamentals of Consolidation

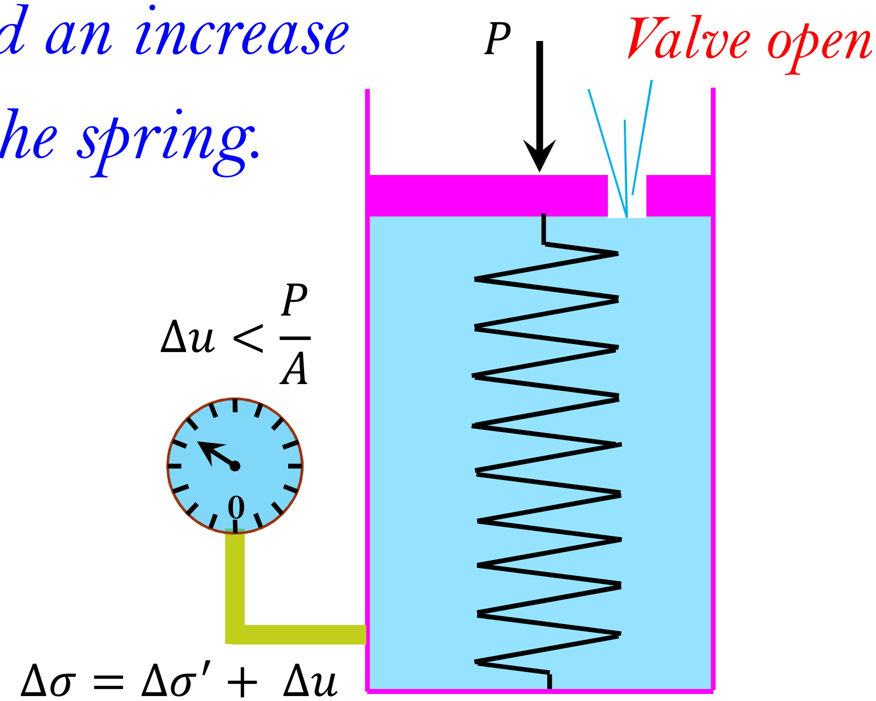
Spring-cylinder model

If the valve is opened, the water will flow outward. This flow will be accompanied by a reduction of the excess hydrostatic pressure and an increase in the compression of the spring.

$$\Delta\sigma = \frac{P}{A}$$

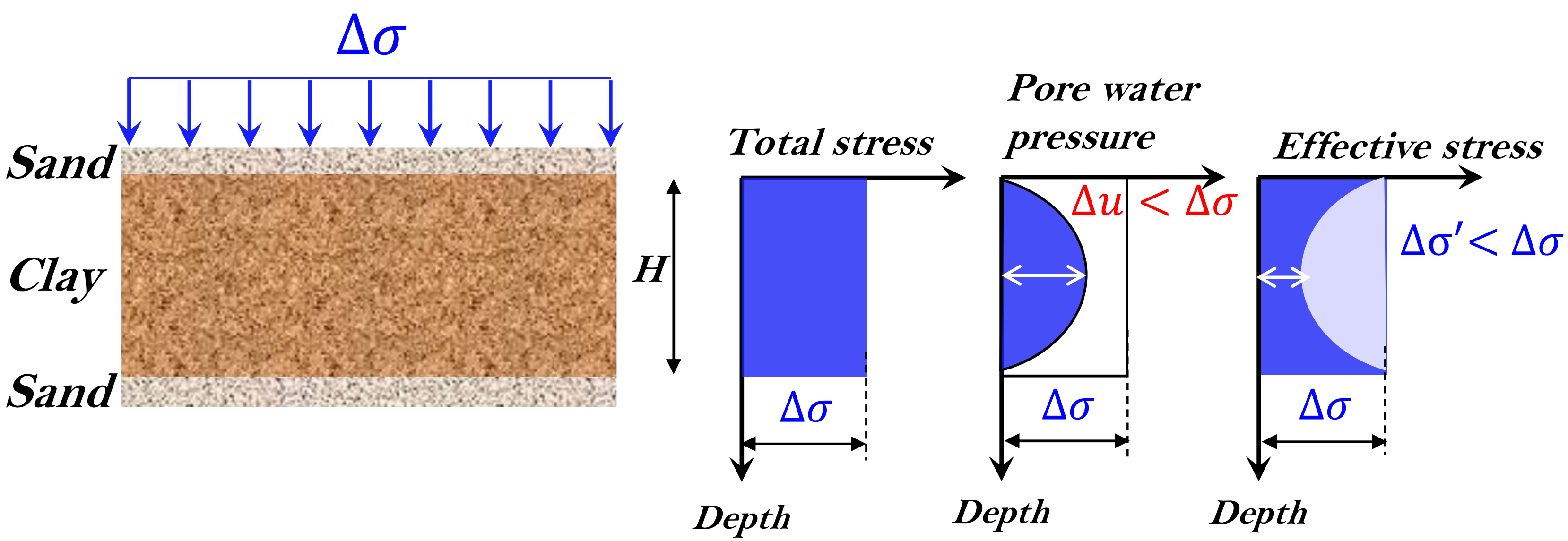
$$\Delta\sigma = \Delta\sigma' + \Delta u$$

$\Delta\sigma' > 0$ *and* $\Delta u < \frac{P}{A}$



Fundamentals of Consolidation

Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress $\Delta\sigma$ *:*

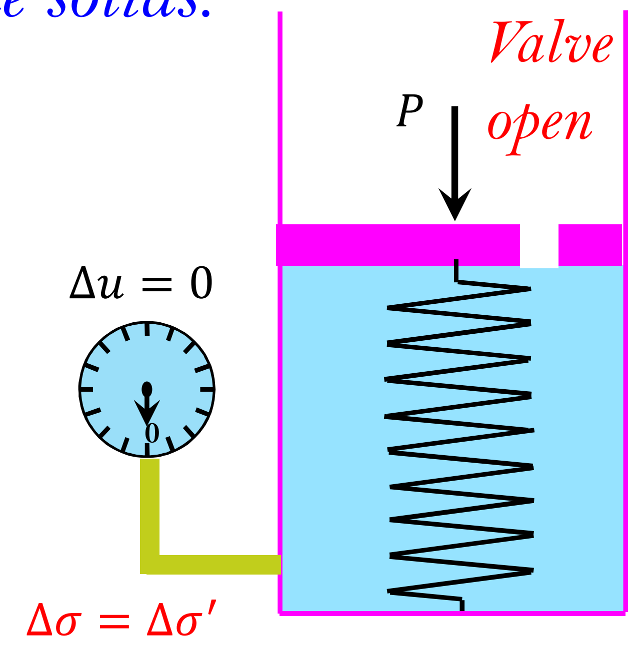


At time $0 < t < \infty$

Fundamentals of Consolidation

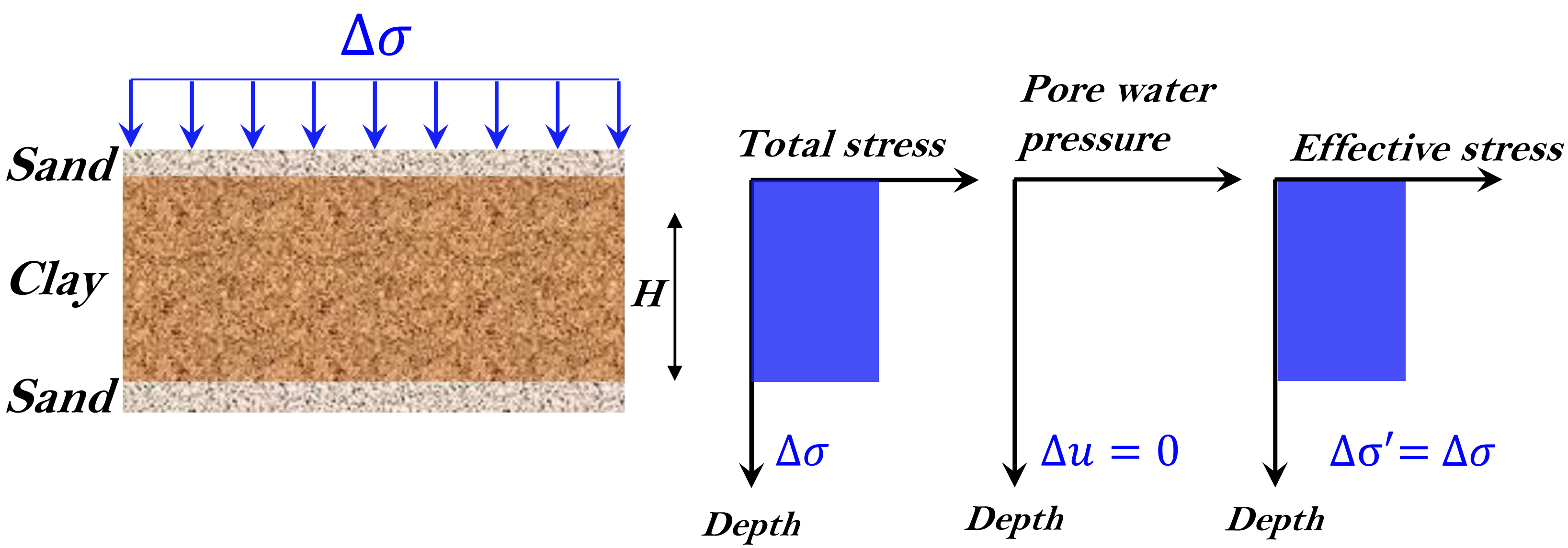
Spring-cylinder model

The spring is totally compressed with final value and the load carried by water therefore now is zero and the entire load is carried by the solids.



Fundamentals of Consolidation

Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress $\Delta\sigma$ *:*



At time $= \infty$

Consolidation Test

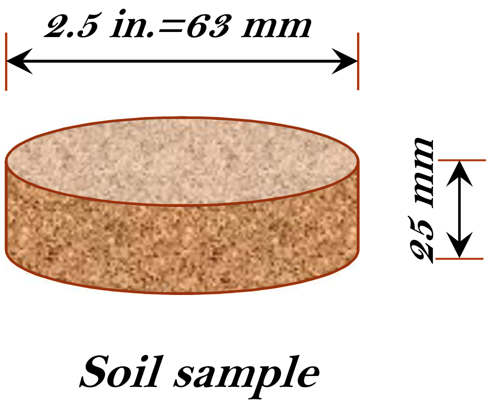
Laboratory consolidation test

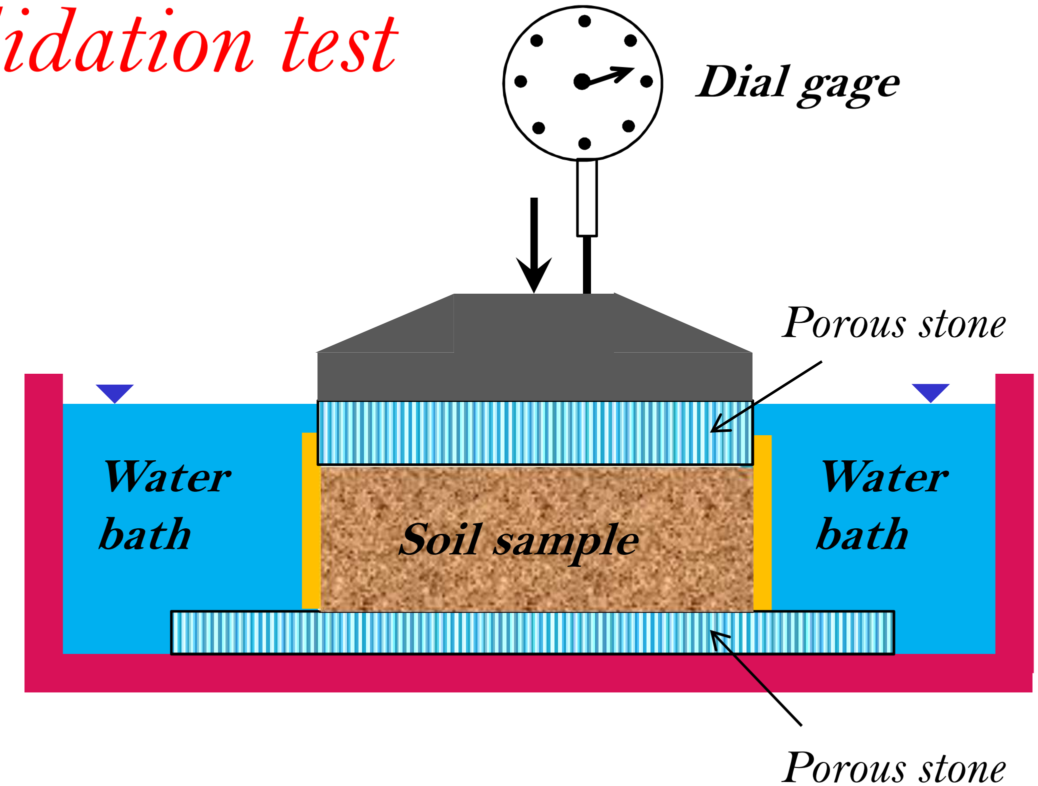
The oedometer test is used to investigate the 1-D consolidation behaviour of fine-grained soils.

1.Place sample in ring
2.Apply load
3.Measure height change
4.Repeat for new load.

Consolidation Test

Laboratory consolidation test

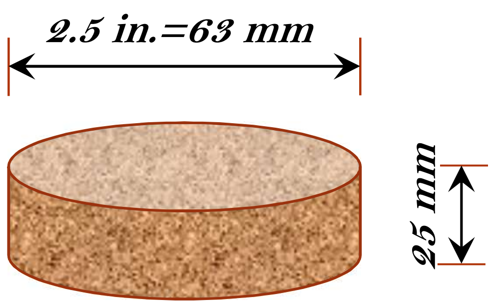




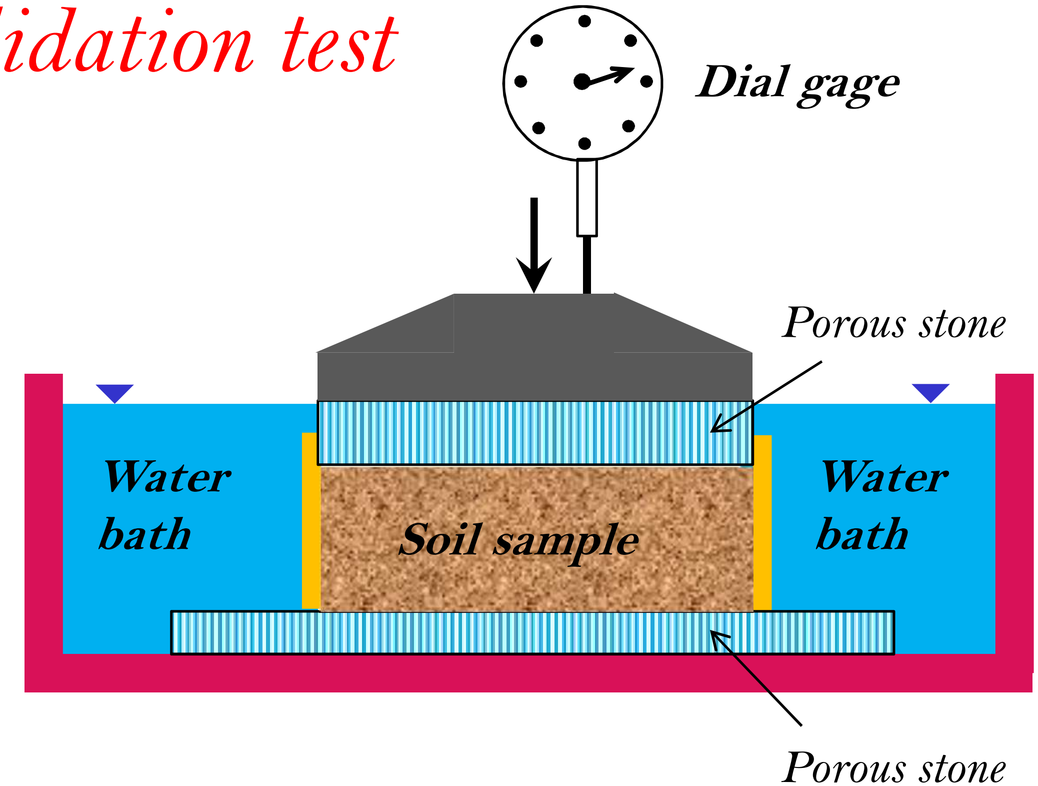
An undisturbed soil sample 25 mm in height and 75 mm in diameter is confined in a steel confining ring and immersed in a water bath.

Consolidation Test

Laboratory consolidation test



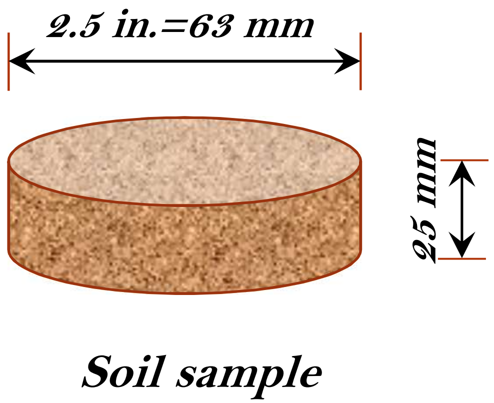
Soil sample

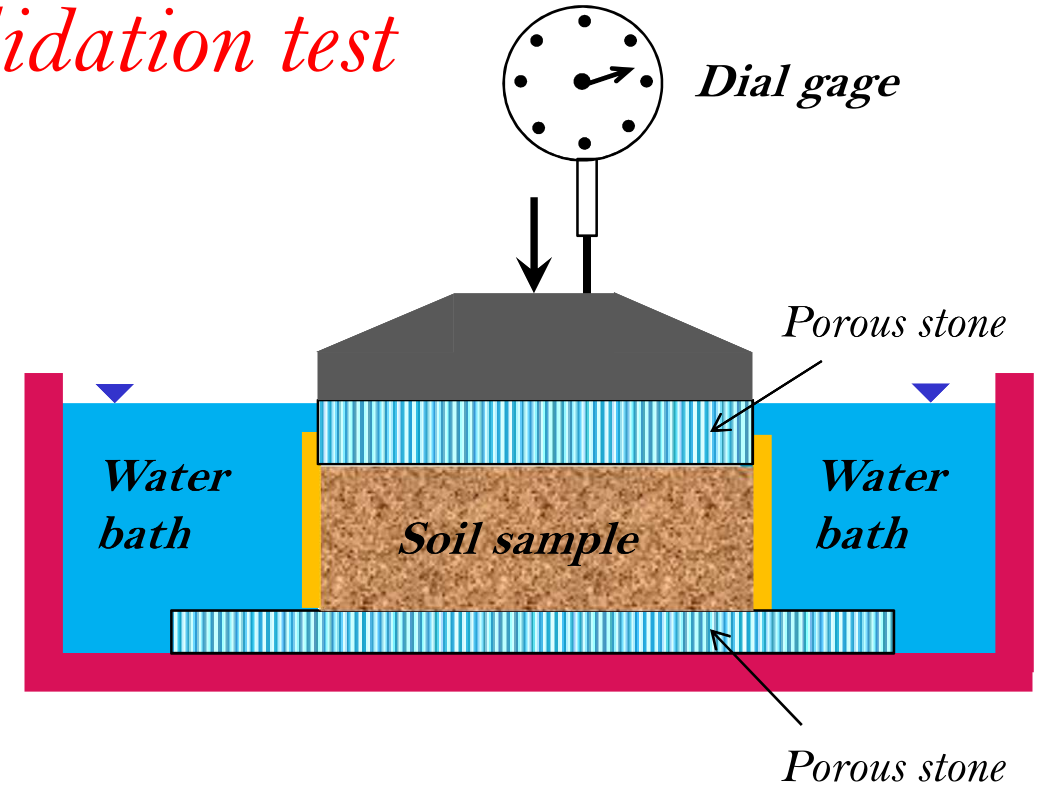


It is subjected to a compressive stress by applying a vertical load, which is assumed to act uniformly over the area of the soil sample.

Consolidation Test

Laboratory consolidation test

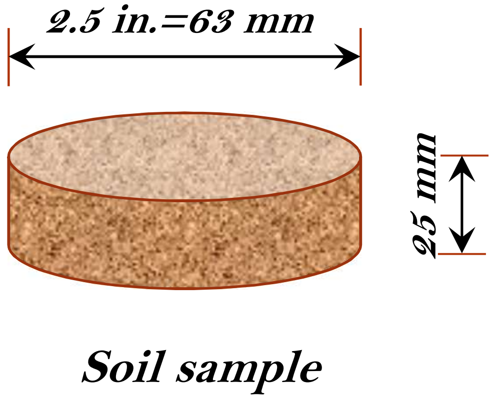


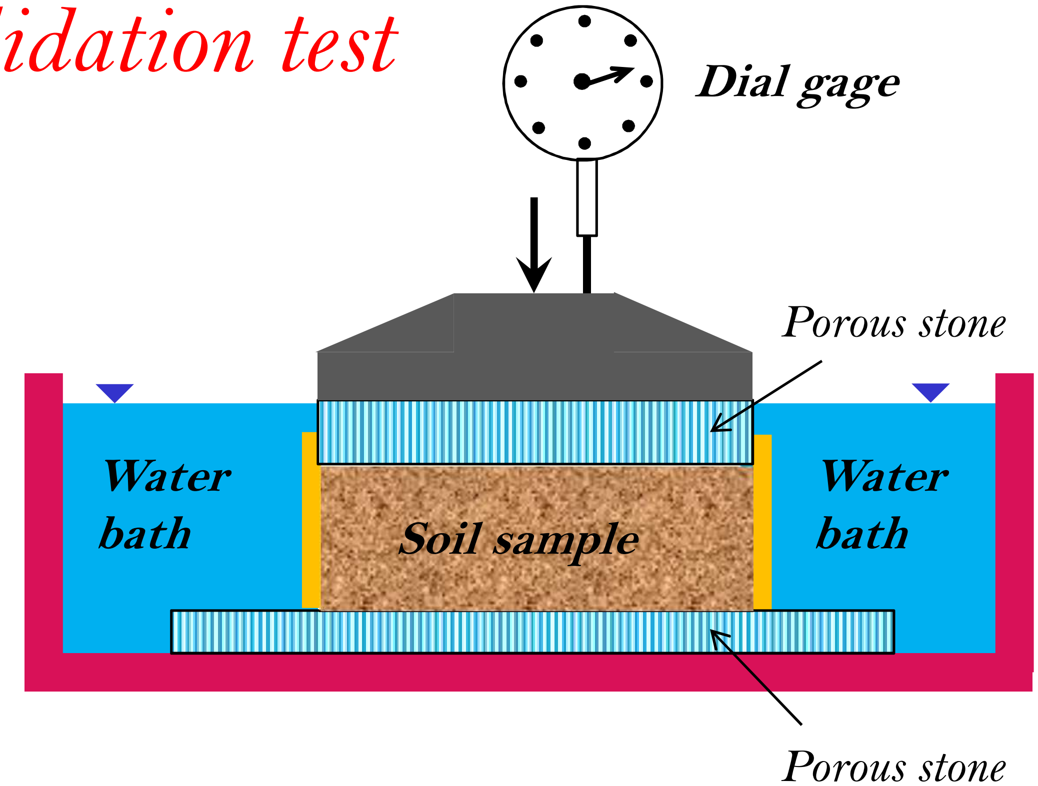


Several increments of vertical stress are applied usually by doubling the previous increment.

Consolidation Test

Laboratory consolidation test

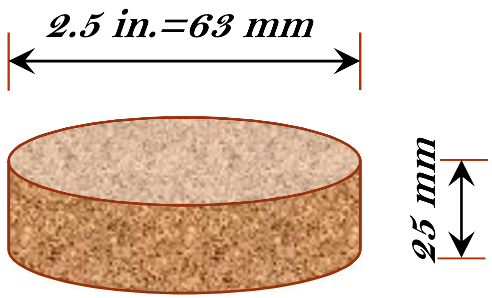




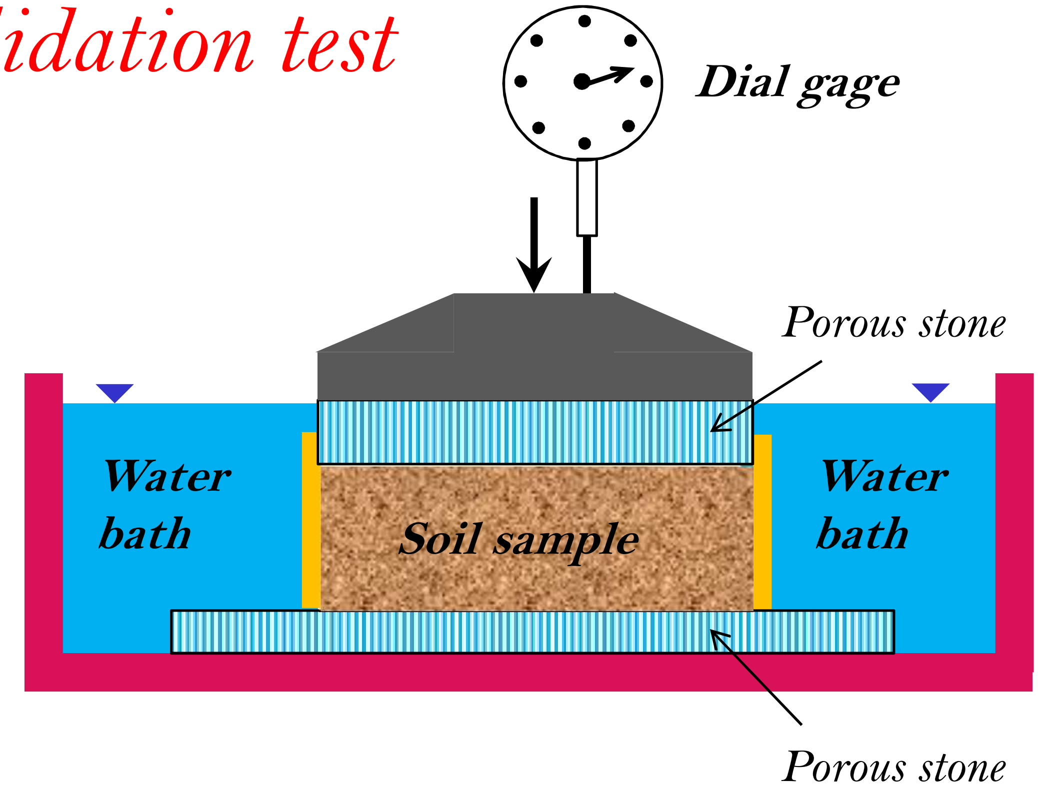
Two-way drainage is permitted through porous disks at the top and bottom

Consolidation Test

Laboratory consolidation test



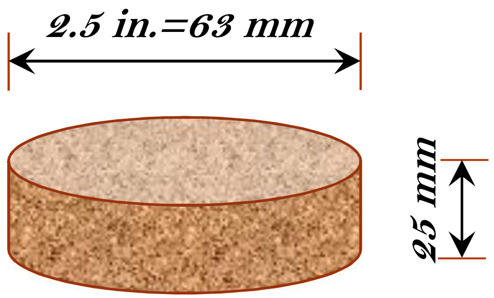
Soil sample

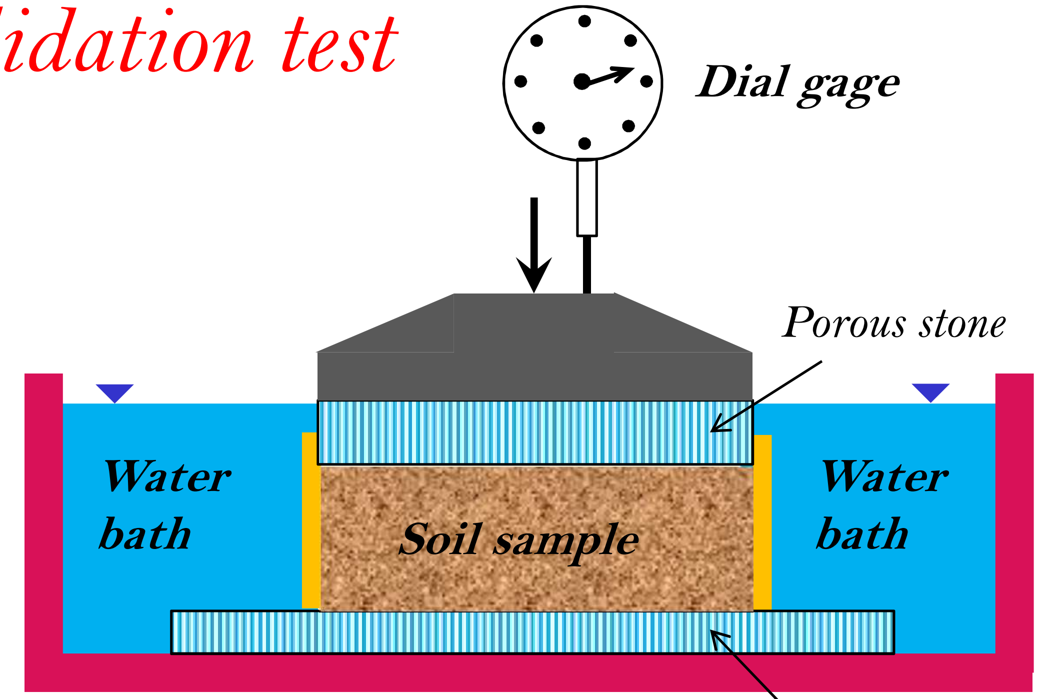


The vertical compression of the soil sample is recorded using highly accurate dial gauges.

Consolidation Test

Laboratory consolidation test





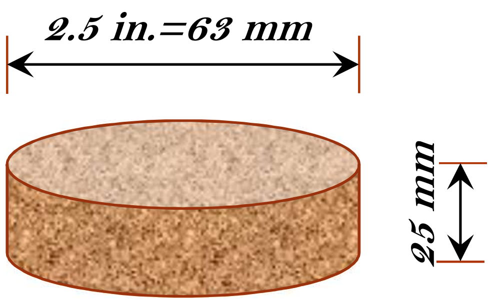
For each increment, the final settlement of the soil sample as well as the time taken to reach the final settlement is recorded.

Consolidation Test

Laboratory consolidation test

Assumption:

- *Load distribution-uniform*
- *Stress distribution(in different height)-the same*
- *No lateral deformation*
- *The area of the sample section-unchangeable*
- *Solid soil-uncompressible*



Soil sample

Consolidation Test

Laboratory consolidation test

A laboratory consolidation test is performed on an undisturbed sample of a cohesive soil to determine its compressibility characteristics. The soil sample is assumed to be representing a soil layer in the ground.

A conventional consolidation test is conducted over a number of load increments. The number of load increments should cover the stress range from the initial stress state of the soil to the final stress state the soil layer is expected to experience due to the proposed construction.

Consolidation Test

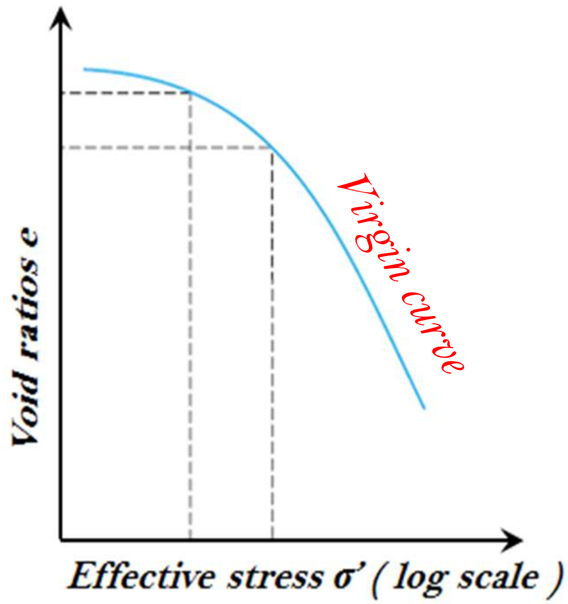
- *Determine the height of solids (Hs) of the specimen in the mold*
- *Determine the change in height (ΔH)*
- *Determine the final specimen height, $H_{t(f)}$*
- *Determine the height of voids (Hv)*
- *Determine the final void ratio*

$$H_s = \frac{W_s}{\left(\frac{\pi}{4}D^2\right)G_s\ \rho_w} \qquad H_v = H_{t(f)} - H_s \qquad e = \frac{H_v}{H_s}$$

Consolidation Test

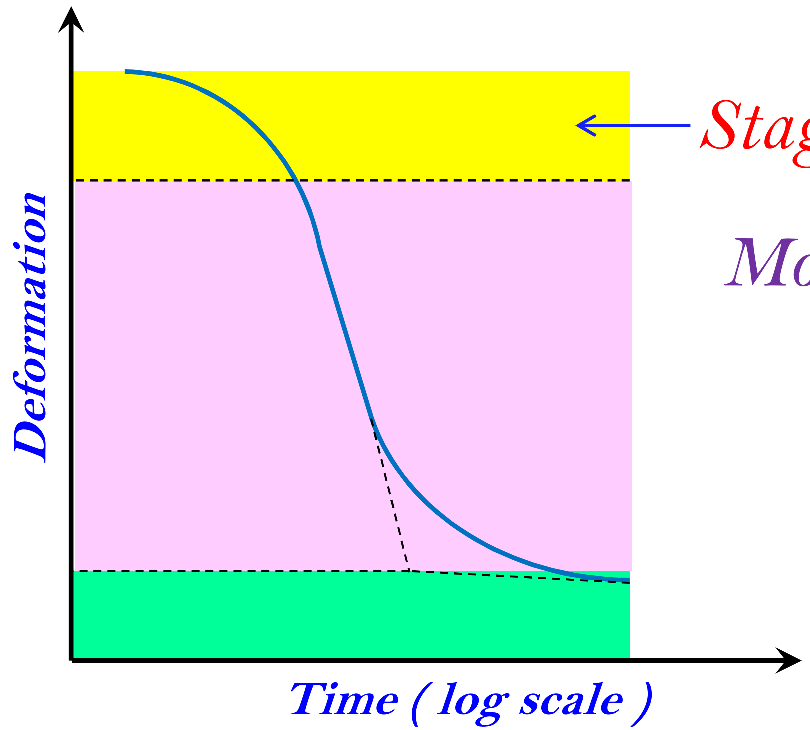
The effective stress σ' and the corresponding void ratios e at the end of consolidation are plotted on semi-logarithmic graph:

In the initial phase, relatively great change in pressure only results in less change in void ratio e. The reason is part of the pressure got to compensate the expansion when the soil specimen was sampled. In the following phase e changes at a great rate



Consolidation Test

The general shape of the plot of deformation of the specimen against time for a given load increment is shown below. From the plot, we can observe three distinct stages:

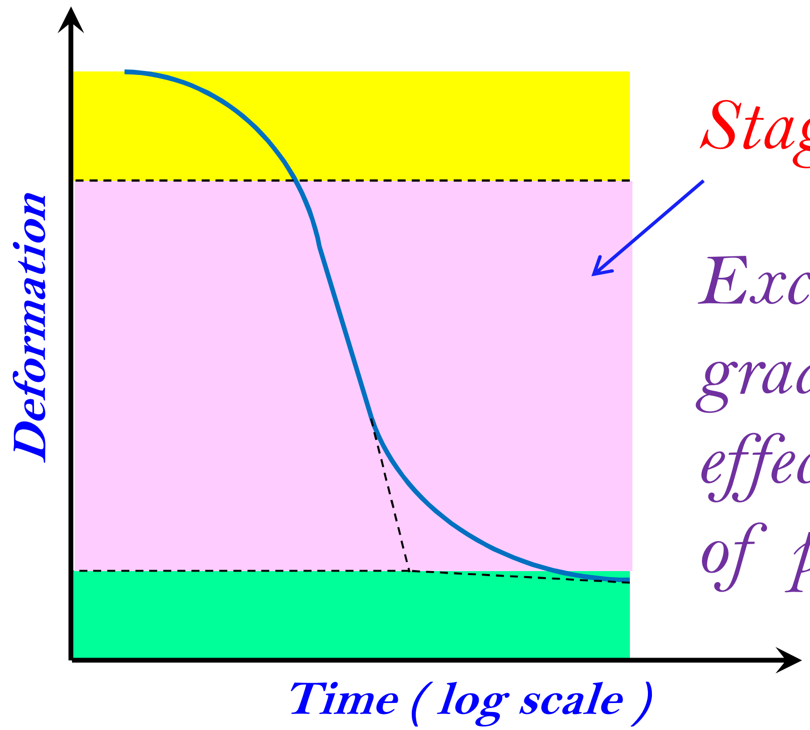


Stage 1: initial Compression

Mostly caused by preloading

Consolidation Test

The general shape of the plot of deformation of the specimen against time for a given load increment is shown below. From the plot, we can observe three distinct stages:

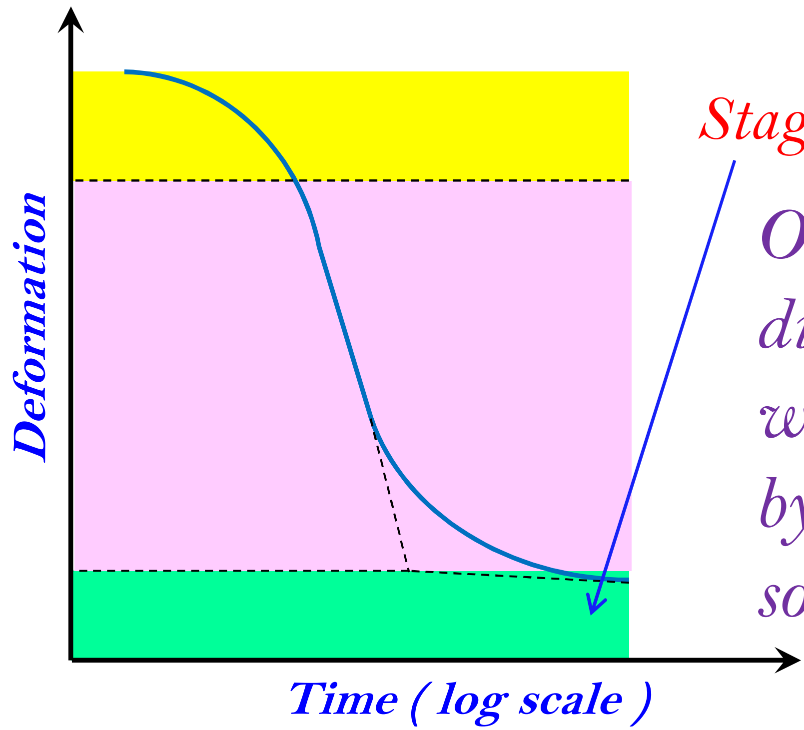


Stage 2: Primary Consolidation

Excess pore water pressure is gradually transferred into effective stress by the expulsion of pore water

Consolidation Test

The general shape of the plot of deformation of the specimen against time for a given load increment is shown below. From the plot, we can observe three distinct stages:



Stage 3: Secondary Consolidation

Occur after complete dissipation of the excess pore water pressure, this is caused by the plastic adjustment of soil fabric

Consolidation Test

Laboratory consolidation test

Increments in a conventional consolidation test are generally of 24 hr. duration and the load is doubled in the successive increment.

The main purpose of consolidation tests is to obtain soil data which is used in predicting the rate and amount of settlement of structures founded on clay.

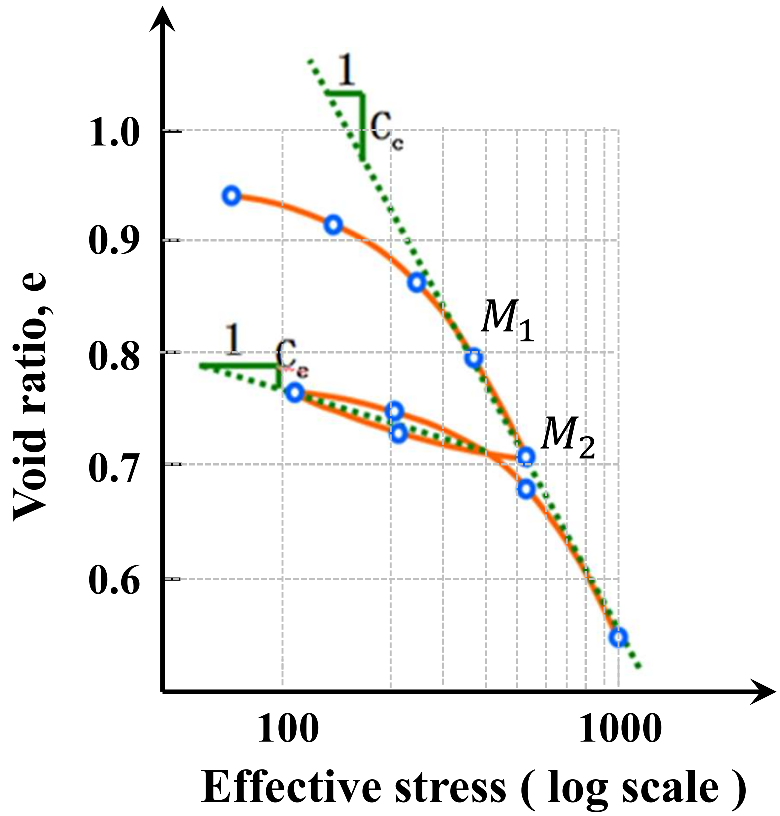
Compressibility Parameters

The most important soil properties determined by a consolidation test are:

1. *Compression Index Cc*

$$C_c = \frac{e_1 - e_2}{\log \sigma_2 - \log \sigma_1}$$

2. *Swelling Index Cs*



Compressibility Parameters

3. Compressibility Coefficient a_v

$$a_v = -\frac{\Delta e}{\Delta \sigma} = \frac{e_1 - e_2}{\sigma_2 - \sigma_1}$$

$a_v < 0.1\ MPa^{-1}$

Low compressibility

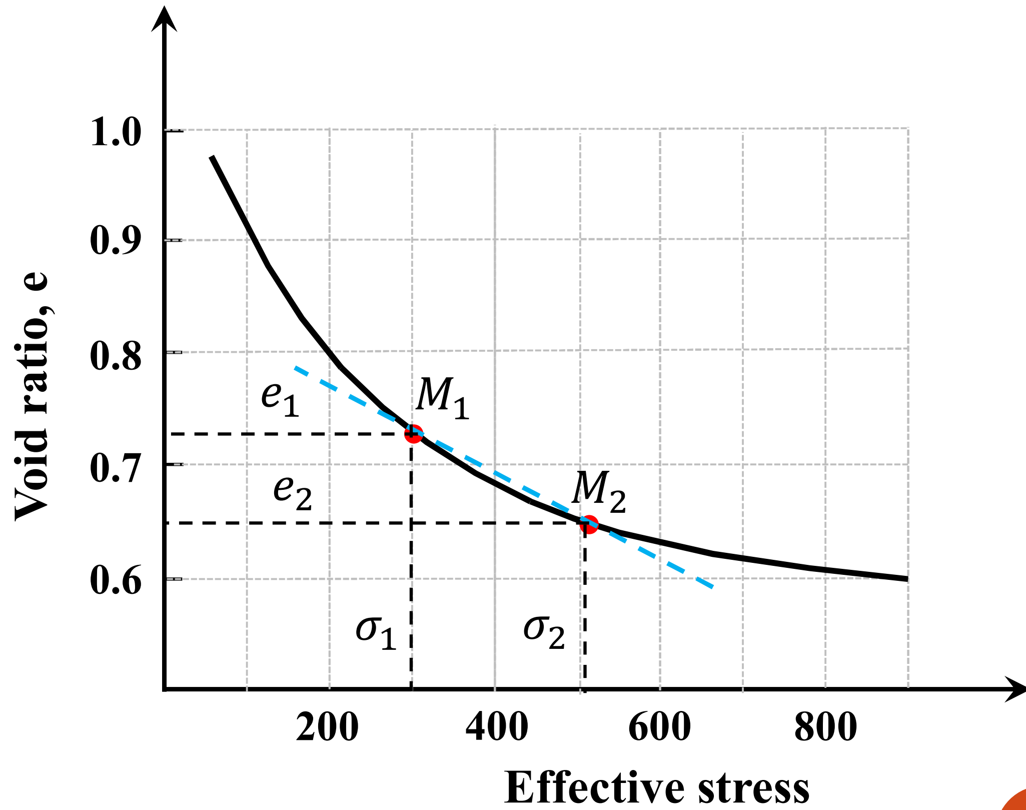
$0.1 \le a_v \le 0.5\ MPa^{-1}$

Middle compressibility

$a_v > 0.5\ MPa^{-1}$

High compressibility

Evaluation of compression with a_v

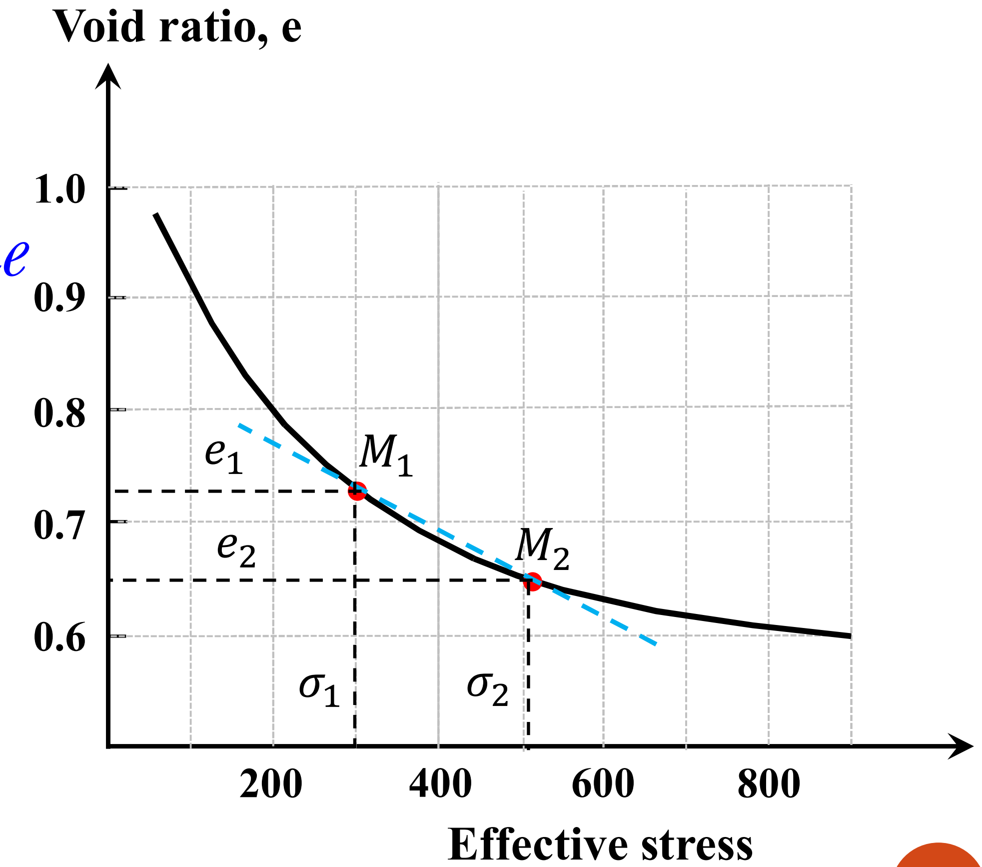


Compressibility Parameters

4. Coefficient of volume compressibility mv

This parameter is defined as change in volume per unit volume as a ratio with respect to the change in stress.

$$m_v = \frac{a_v}{1 + e_o}$$



Compressibility Parameters

5. Preconsolidation Pressure

Normally consolidated clay, whose present effective overburden pressure is the maximum pressure that the soil was subjected to in the past.

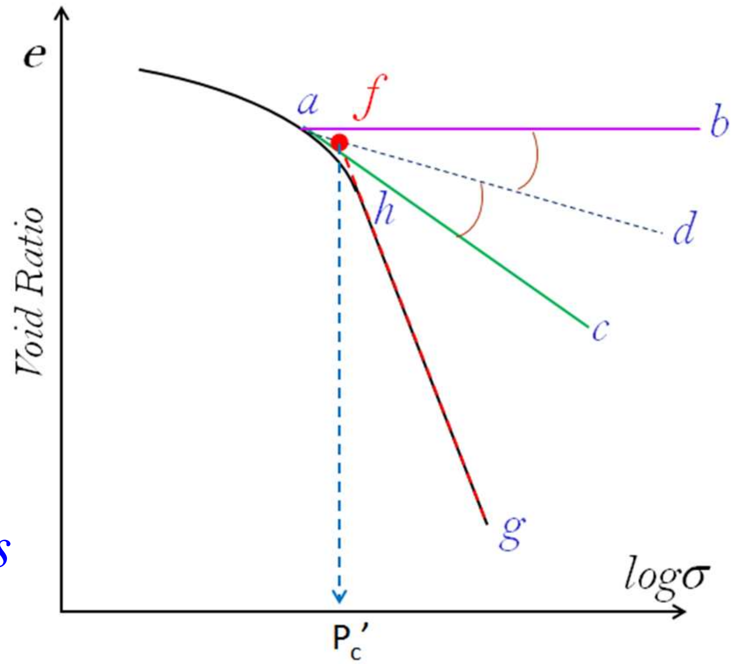
Overconsolidated, whose present effective overburden pressure is less than that which the soil experienced in the past. The maximum effective past pressure is called the preconsolidation pressure.

Compressibility Parameters

5. Preconsolidation Pressure

Preconsolidation pressure can be determined as follow :

1. *Establish point a, at which curve has a minimum radius of curvature.*
2. *Draw a horizontal line ab.*
3. *Draw the line ac tangent at a.*
4. *Draw the line ad, which is the bisector of the angle bac.*
5. *Project the straight-line portion gh of the e–log σ' plot back to intersect line ad at f. The abscissa of point f is the preconsolidation pressure, .*



Compressibility Parameters

5. Preconsolidation Pressure

The overconsolidation ratio (OCR) *for a soil can now be defined as*

$$OCR = \frac{P_c'}{\sigma_c'}$$

where :

P_c' = *preconsolidation pressure*

σ_c' = *present effective vertical pressure*

The OCR for an OC soil is greater than 1.

Most OC soils have fairly high shear strength.

The OCR cannot have a value less than 1.

Compressibility Parameters

6. Coefficient of consolidation Cv

The rate of consolidation settlement is estimated using the Coefficient of consolidation Cv. This parameter is determined for each load increment in the test.

The coefficient of consolidation (c_v) can be determined by the (Casagrande) Logarithm-of-Time and by (Taylor) Square –Root of Time Methods.

Compressibility Parameters

6. Coefficient of consolidation Cv

Logarithm –of – time Method

The following construction are needed to determine Cv:

1. Extend the straight line portions of primary and secondary consolidations to intersect at A. The ordinate of A is represent by d100 - that is, the deformation at the end of 100% primary consolidation

Compressibility Parameters

6. Coefficient of consolidation Cv

Logarithm –of – time Method

The following construction are needed to determine Cv:

2. The initial curved portion on the plot of deformation versus logt is approximated to be a parabola on the natural scale. Select times t_1 and t_2 on the curved portion such that $t_2 = 4\ t_1$. Let the difference of specimen deformation during time ($t_2 - t_1$) be equal to x

Compressibility Parameters

6. Coefficient of consolidation Cv

Logarithm –of – time Method

The following construction are needed to determine Cv:

3. Draw a horizontal line DE such that the vertical distance BD is equal to x.

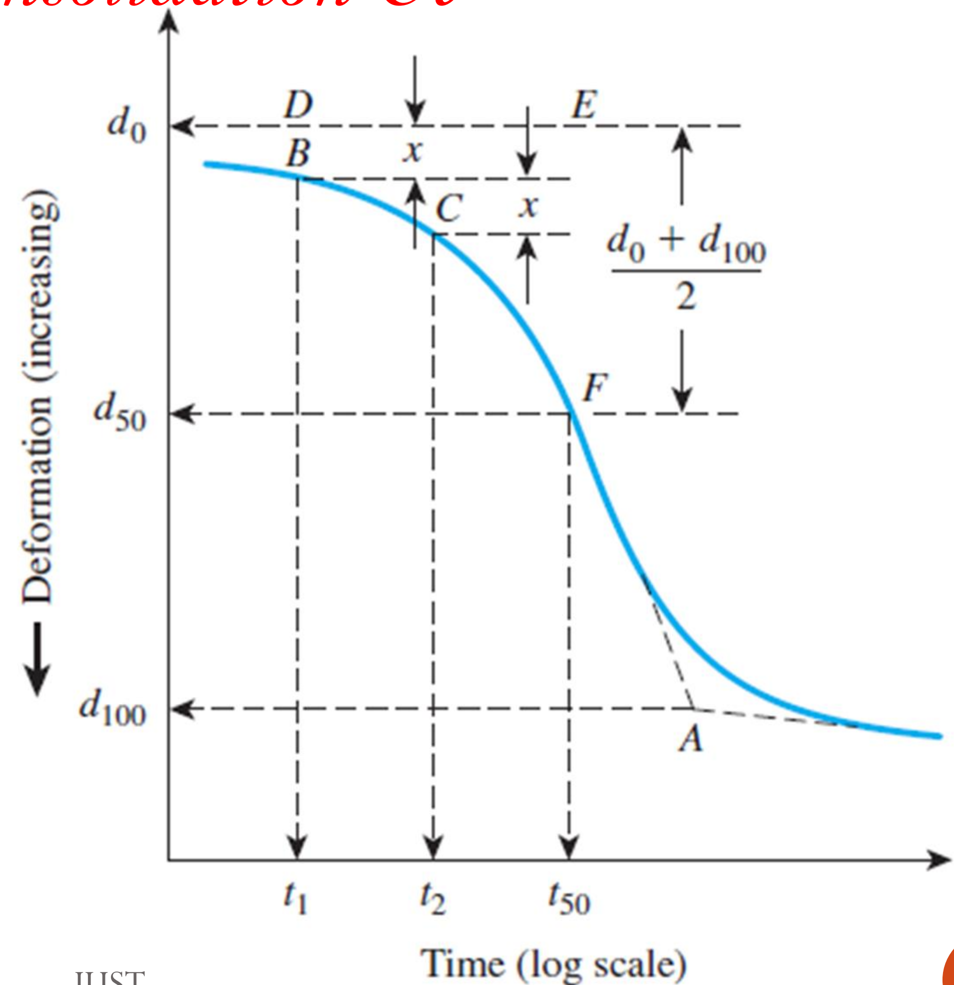
The deformation corresponding to the line DE is d_0 (that is deformation at 0% consolidation

4. The ordinate of point F on the consolidation curve represent the deformation at 50% primary consolidation and its abscissa represent the corresponding time (t_{50})

Compressibility Parameters

6. Coefficient of consolidation Cv

Logarithm –of – time Method



Compressibility Parameters

6. Coefficient of consolidation Cv

Logarithm –of – time Method

The following construction are needed to determine Cv:

5. For 50% average degree of consolidation Tv = 0.197, so

$$c_v = \frac{0.197\, H_{dr}^2}{t_{50}}$$

where H_{dr} = average longest drainage path during consolidation.

Compressibility Parameters

6. Coefficient of consolidation Cv

Logarithm –of – time Method

The following construction are needed to determine Cv:

For specimen drained at both top and bottom, H_{dr} equals one-half the overage height of the specimen during consolidation .

For specimen drained on only one side, H_{dr} equals the average height of the specimen during consolidation.

Compressibility Parameters

6. Coefficient of consolidation Cv

Square-Root-of-Time Method (Taylor)

Plot a deformation against the square root of time

1. Draw a line AB through the early portion of the curve
2. Draw a line AC such that OC = 1.15 OB.

The abscissa of point D, which is the intersection of AC and the consolidation curve, gives the square root of time for 90% consolidation

Coefficient of Consolidation

6. Coefficient of consolidation Cv

Square-Root-of-Time Method (Taylor)

Plot a deformation against the square root of time

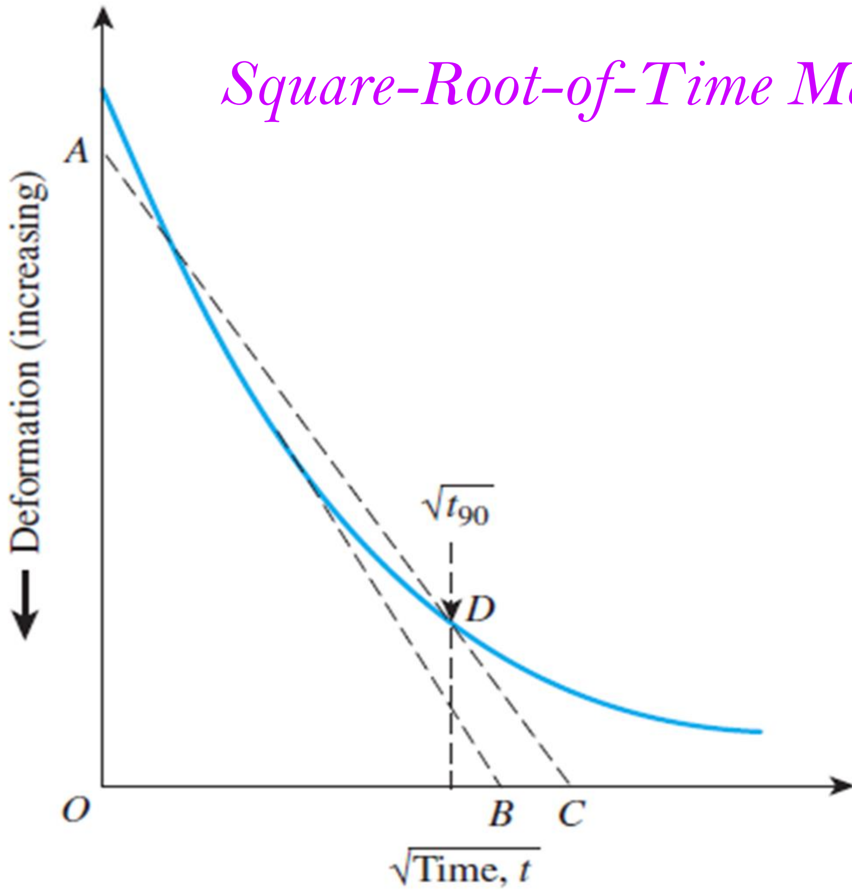
3. For 90% consolidation $T_{90} = 0.848$, so

$$c_v = \frac{0.848\, H_{dr}^2}{t_{90}}$$

Coefficient of Consolidation

6. Coefficient of consolidation Cv

Square-Root-of-Time Method (Taylor)



Consolidated Settlement

Primary consolidation

For normally consolidated clay

$$S = \frac{c_c \ h}{1 + e_o} log\left(\frac{\sigma'_{vo} + \ \Delta\sigma_v}{\sigma'_{vo}}\right)$$

For overconsolidated clay with $\sigma'_{vo} + \ \Delta\sigma_v \leq P'_c$

$$S = \frac{c_s \ h}{1 + e_o} log\left(\frac{\sigma'_{vo} + \ \Delta\sigma_v}{\sigma'_{vo}}\right)$$

where $C_c = compression\ index$

$C_s = swelling\ index$

Consolidated Settlement

Primary consolidation

For overconsolidated clay with

$$\sigma'_{vo} \leq P'_c \leq \sigma'_{vo} + \Delta\sigma_v$$

$$S = \frac{c_s \; h}{1+e_o} log\left(\frac{P'_c}{\sigma'_{vo}}\right) + \frac{c_c \; h}{1+e_o} log\left(\frac{\sigma'_{vo} + \Delta\sigma_v}{P'_c}\right)$$

where

$e_o = initial\ void\ ratio\ of\ the\ clay\ layer$

$P'_c = preconsolidation\ pressure$

$h = thickness\ of\ the\ clay\ layer$

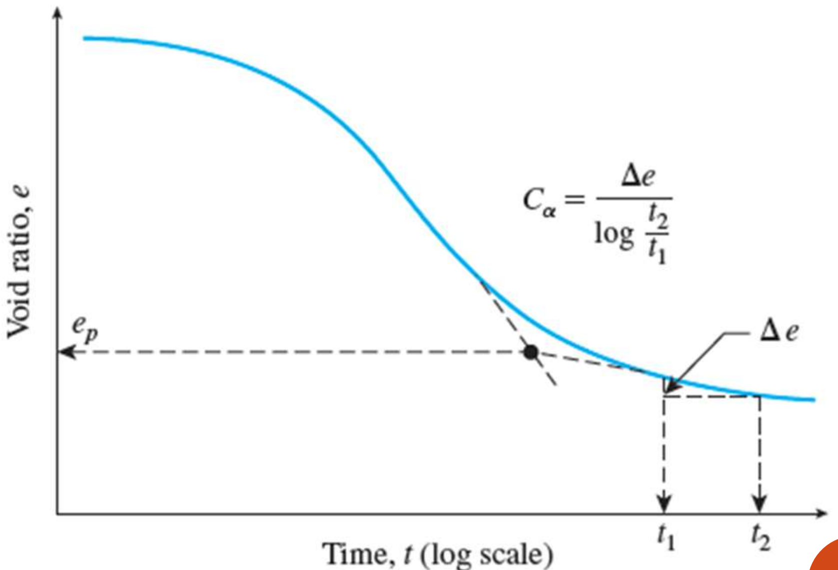
$\sigma'_{vo} = overburden\ effective\ pressure$
$at\ the\ middle\ of\ the\ clay\ layer$

Consolidated Settlement

Secondary Consolidation

At the end of primary consolidation (i.e., after the complete dissipation of excess pore water pressure) some settlement is observed that is due to the plastic adjustment of soil fabrics. This stage of consolidation is called secondary consolidation.

A plot of deformation against the logarithm of time during secondary consolidation is practically linear as shown in Figure.



Consolidated Settlement

Secondary Consolidation

The secondary compression index can be defined as:

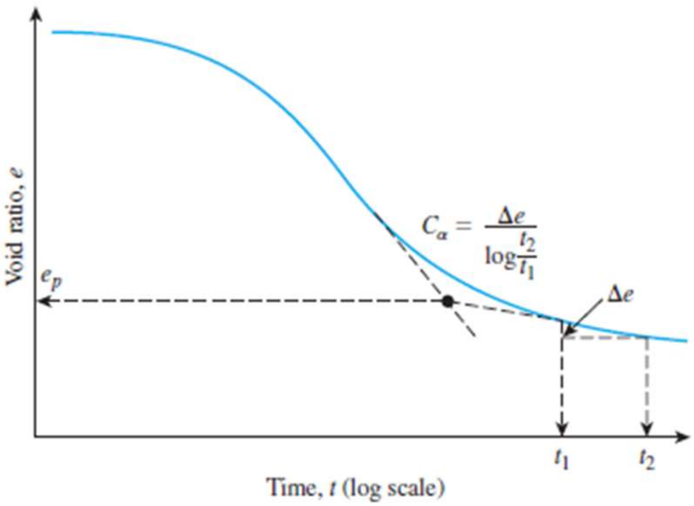
$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1}$$

where

C_α = secondary compression index

Δe = change of void ratio

t_1 and t_2 = time



Consolidated Settlement

Secondary Consolidation

The magnitude of the secondary consolidation can be calculated as :

$$S_{cs} = C'_{\alpha} H \, log(t_2/t_1)$$

where

$$C'_{\alpha} = \frac{C_{\alpha}}{1 + e_p}$$

$e_p = void\ ratio\ at\ the\ end\ of\ primary\ consolidation$

$H = thickness\ of\ the\ clay\ layer$

Consolidated Settlement

Secondary Consolidation

Secondary consolidation settlement is more important in the case of all organic and highly compressible inorganic soils.
In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

Time Rate of Consolidation

Terzaghi(1925) derived the time rate of consolidation based on the following assumptions:

- *1 The soil is homogeneous and fully saturated.*
- *2 There is a unique relationship, independent of time, between void ratio and effective stress.*
- *3 The solid particles and water are incompressible.*
- *4 Compression and flow are one-dimensional (vertical).*
- *5 Strains in the soil are relatively small.*
- *6 Darcy's law is valid at all hydraulic gradients.*
- *7 The coefficient of permeability and volume compressibility remain constant throughout the process .*

Time Rate of Consolidation

Degree of consolidation

The average degree of consolidation for the entire depth of the clay layer at any time t can be expressed as

$$U = \frac{S_t}{S_f} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z\,dz}{u_o}$$

where

$U =$ average degree of consolidation

$S_t =$ settlement of the layer at time t

$S_f =$ final settlement of the layer from primary consolidation

Time Rate of Consolidation

Degree of consolidation

The values of the time factor and their corresponding average degrees of consolidation for the case presented in may also be approximated by the following simple relationship:

$$For\ U = 0\ to\ 60\%,\quad T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$$For\ U > 60\%,\quad T_v = 1.781 - 0.933 log(100 - U\%)$$

$$T_v = \frac{C_v\, t}{H_{dr}^2} \qquad c_v = \frac{k(1+e_1)}{a\gamma_\omega}$$

Worked Example

Example 1

The results of laboratory consolidation test on a clay sample are given below:

Pressure, kN/m^2	23.94	47.88	95.76	191.52	383.04	766.08
Void ratio , e	1.112	1.105	1.080	0.985	0.850	0.731

1. Draw an e-logσ plot
2. Determine the preconsolidation pressure.
3. Find the compression index, Cc.

Worked Example

Example 2

Data obtained from one increment in a conventional multi increment consolidation test :

Time Elapsed (min)	0.00	0.25	0.5	1.0	2.0	4.0	8.0	15.0	30	60	120	1440
Dial Reading (mm)	3.74	3.86	3.88	3.92	3.99	4.08	4.19	4.29	4.37	4.41	4.44	4.52

Dial gauge reading at the start of the current increment = 3.744 mm

Initial height of the sample = 20 mm

Specific gravity of the particles = 2.65

Current load increment is from 60 kN/m^2 to 120 kN/m^2.

Required :

The coefficient of consolidation.

Worked Example

Time Elapsed (min)	0.00	0.25	0.5	1.0	2.0	4.0	8.0	15.0	30	60	120	1440
Root time (mm)	0.0	0.5	0.71	1.0	1.41	2.0	2.83	3.87	5.48	7.75	10.95	37.95
Settlement (mm)	0.0	0.12	0.14	0.18	0.25	0.34	0.45	0.55	0.63	0.67	0.70	0.78

Logarithm –of – time Method

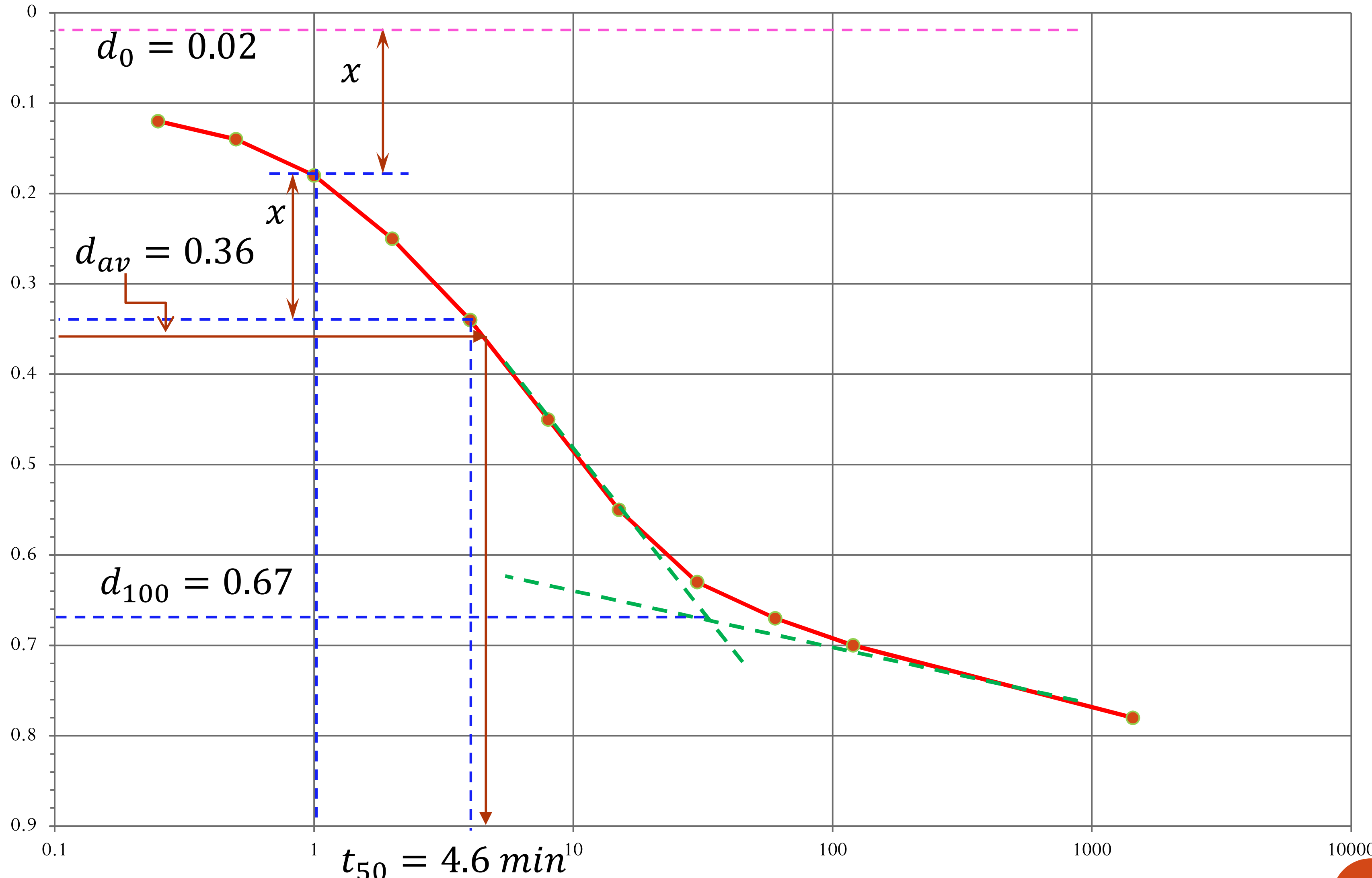
$$h = 20 - 3.74 = 16.26\, mm$$

$$d_0 = 0.02 \qquad d_{100} = 0.67$$

$$d_{av} = \frac{d_0 + d_{100}}{2} = \frac{0.05 + 0.67}{2} = 0.36 \rightarrow \quad t_{50} = 4.6\, min$$

$$c_v = \frac{0.197 * H_{dr}^2}{t_{50}} \qquad c_v = \frac{0.197 * (16.26/2)^2}{4.6} = 2.83\, mm^2/min$$

Worked Example



Worked Example

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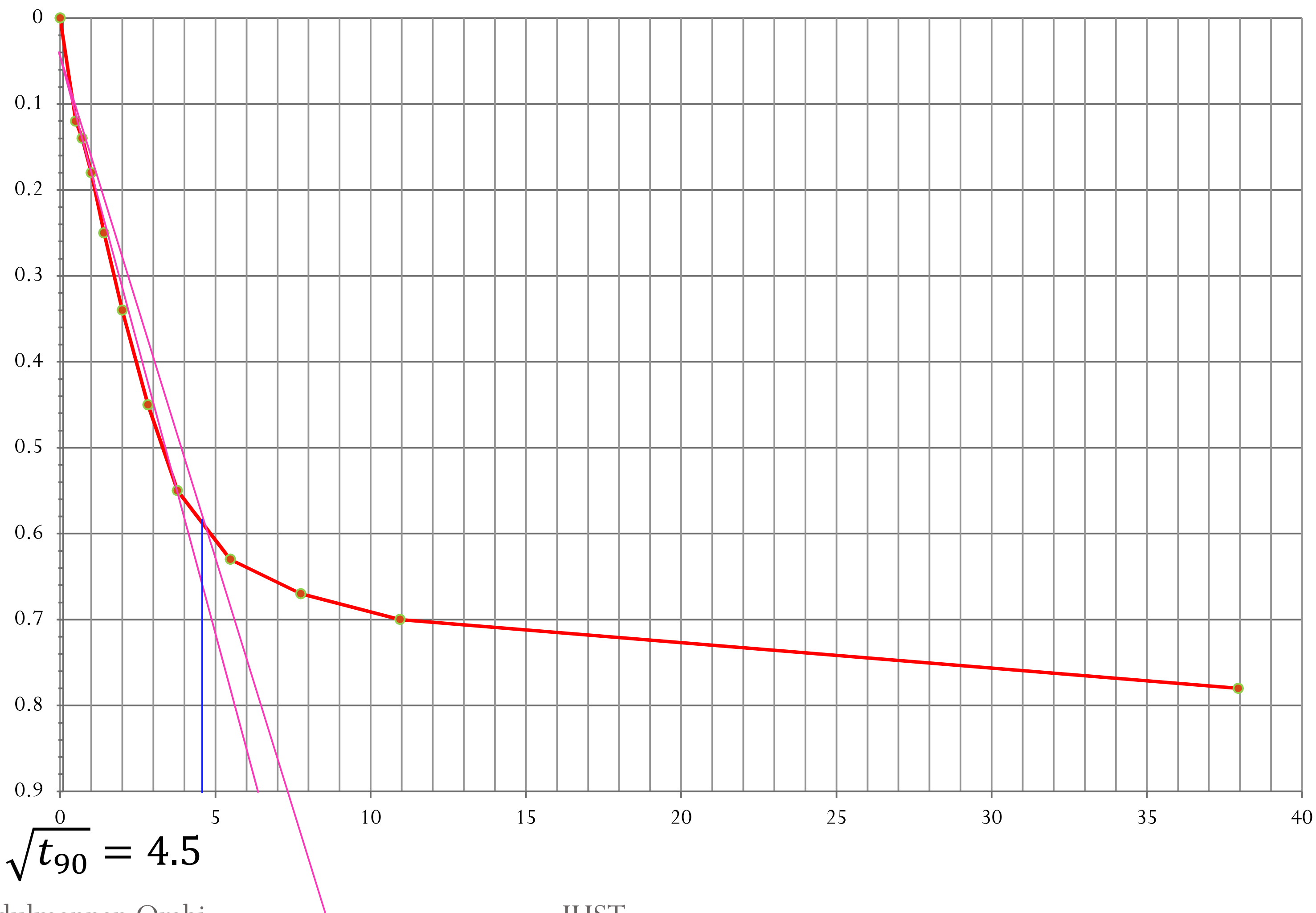
Square-Root-of-Time Method (Taylor)

$$\sqrt{t_{90}} = 4.5 \qquad t_{90} = 20.25\ min$$

$$c_v = \frac{0.848 * H_{dr}^2}{t_{90}} \qquad c_v = \frac{0.848 * (16.26/2)^2}{20.25} = 2.77\ mm^2/min$$

Worked Example

Square-Root-of-Time Method (Taylor)



$$\sqrt{t_{90}} = 4.5$$

End of the Lecture

Let Learning Continue